OPTIMAL RECOVERY FROM DISRUPTIONS IN WATER DISTRIBUTIONS NETWORKS

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By

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Abstract:

In this study, we develop a bi-level optimization model for recovery of a disrupted water distribution system. The model minimizes the total cost of recovery of the system. The cost includes the repair cost of the system and systemic impact of disruption on the system during the repair process. The systemic cost is calculated in terms of the unmet demand while the system is still damaged. The upper level problem is to schedule the repair tasks, and the lower level problem is to optimize the supply of water (optimal mitigation) given the schedule from upper level problem. The upper level problem is solved using Simulated Annealing, and the lower level problem uses a Generalized Reduced Gradient algorithm. We first apply and validate the model on a small water distribution system with only three elements damaged due to disruption. In this case, the recovery process requires tasks that can be performed in only one mode. We later apply the model to a larger and more complex water distribution system with eight elements damaged due to disruption. We perform three different experiments on this system. In the first experiment, limited resources are available at each time period. In the second experiment, the resources are increased by 50%, and in third experiment, some tasks are provided with an additional mode. The results show that the availability of resources has a significant impact on total cost of recovery of systems. Adding modes to a few tasks can help in reducing the total cost of disruption on the system.

BIOGRAPHICAL SKETCH

Munir Ahmad Nayak was born in a small village of beautiful mountainous region, Kashmir. Munir did his bachelors in Civil Engineering from National Institute of Technology at Srinagar, a prestigious institute in Kashmir. After his graduation, Munir went for higher studies to Indian Institute of Technology Bombay, at Mumbai, where he completed his Master of Technology in Civil Engineering.

I dedicate this work to my MS thesis adviser, Professor Mark Alan Turnquist, for his extreme kindness and support, and my parents for their continuous encouragement during my work.

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1. Introduction

Water distribution systems are among the most important civil infrastructures in the present world. Our society is hugely dependent on well-functioning water distribution systems and the President's Commission on Critical Infrastructure Protection(CIP) [*Clinton*, 1998] grouped water distribution systems among key infrastructures that are both vital assets and among the potential threats areas. Any disruption in the system can have serious societal and industrial consequences, which may take form of huge financial losses, long-term health issues, and possible life losses. Because of such consequences, a Presidential directive [*Bush*, 2003] focused not only on protection of these systems but also on mitigation, resilience and recovery of the systems in the event of disruption. Disruptions can result from natural disasters like earthquakes, floods, etc. They can also be the result of intentional attacks, which include physical damage to the systems, rendering the systems non-functional, cyber-attacks on systems, and biological and chemical attacks[*Tiemann*, 2010].

Proposed methods of protecting the systems include installation of early warning detectors, biosensors, security cameras, etc. [*Amin et al.*, 2012; *Cho et al.*, 2013; *Ostfeld and Salomons*, 2004; *Raciti et al.*, 2012] Mitigation policies are proposed in order to minimize the consequences of disruption. For example, [*Jeong et al.*, 2006]developed a model to continue supply of water to more important customers while the water distribution network is only partially functional, so that adverse effects can be minimized. [*H. and Abraham*, 2009] developed a scheduling model that schedules water supply in such a way that each customer gets water at some point of time in a day; hence all the customers are at least partially served. [*Turner et al.*, 2012] suggested minimizing the consequences of physical damage on a water distribution network by finding an undamaged sub-network (or residual network) that is hydraulically feasible and hence, can receive water under pressure head. Water distributing trucks can be used to supply water to the rest of the network. The identification of the subset network to be pressurized is based on minimization of water shortage cost and water distribution costs by trucks.

Resilience of water distribution systems is important to make the distribution system robust, resistant, and have least impact due to damage. Resilience is defined and measured in different ways in network problems such as transportation, communication, and water distribution networks. [*Todini*, 2000] used an index of resilience that defines resilience as the intrinsic capability of the system to overcome failures. [*Qiao et al.*, 2007] defined a network element to be resilient if the cost of attacking the element is more than the consequences due to attack. [*Zhuang et al.*, 2012] defined resilience as the ability of a system to recover from a failure to a satisfactory state. Recently [*Turnquist and Vugrin*, 2013; *Vugrin et al.*, 2010] defined resilience of infrastructure systems as the ability of the systems to withstand, adapt to, and rapidly recover from the effects of disruptive event while attempting to continue delivery of critical services. For the present analysis, the definition provided by [*Turnquist and Vugrin*, 2013]is adopted.

An important dimension of infrastructure resilience is the recovery of the system after damage. The present study focuses on optimizing recovery of damaged water distribution systems based on a broad perspective of system resilience provided by [*Turnquist and Vugrin*, 2013; *Vugrin et al.*, 2010]. In this perspective, the recovery process should be scheduled in such a way that minimizes the overall cost of damage. The cost includes the Total Resource Expenditure (TRE) incurred in recovery of the system to its original state, and the cost incurred due to impact of damage on the system, Systemic Impact (SI). This perspective on resilience is illustrated generically in Figure 1. Some system performance measure, F, has a nominal value (i.e., under normal operating conditions) F_0. At some time t_0, the system is operating normally and then suffers a disruption. The disruption causes a rapid deterioration in system performance to some level F_{min} at time t_1. Recovery then begins and the system performance returns to normal at a later time t_2. The diagram in Figure 1 is illustrative only, and is not drawn to scale. For example, the time over which deterioration occurs ($t_1 - t_0$) may be very short and the time of recovery ($t_2 - t_1$) may be quite long, and the amount of performance degradation ($F_0 - F_{min}$) is very context-dependent. A useful measure of SI for this system is $Q_1 = \int_{t_0}^{t_2} [F_0 - F(t)]dt$.

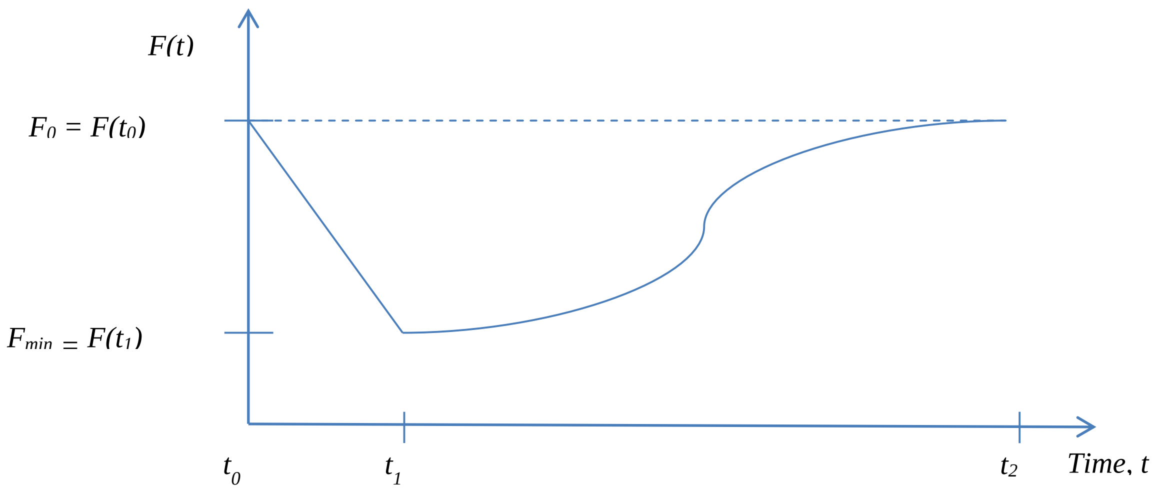

Figure 1 Generic concept of disruption and recovery

Concern with both the restoration of system performance and the resource expenditure required to do so is illustrated in Figure 2. As illustrated in Figure 2, these measures of SI (e.g. Q_1) can be affected by the choice of recovery plans (different actions of different costs), and this is measured as a change in TRE. Thus, in this framework, the focus of attention in system recovery is on a composite measure:

$$Z = SI + \alpha TRE$$

where α is a weighing factor that serves for both unit conversion and relative weighting between SI and TRE in overall evaluation.

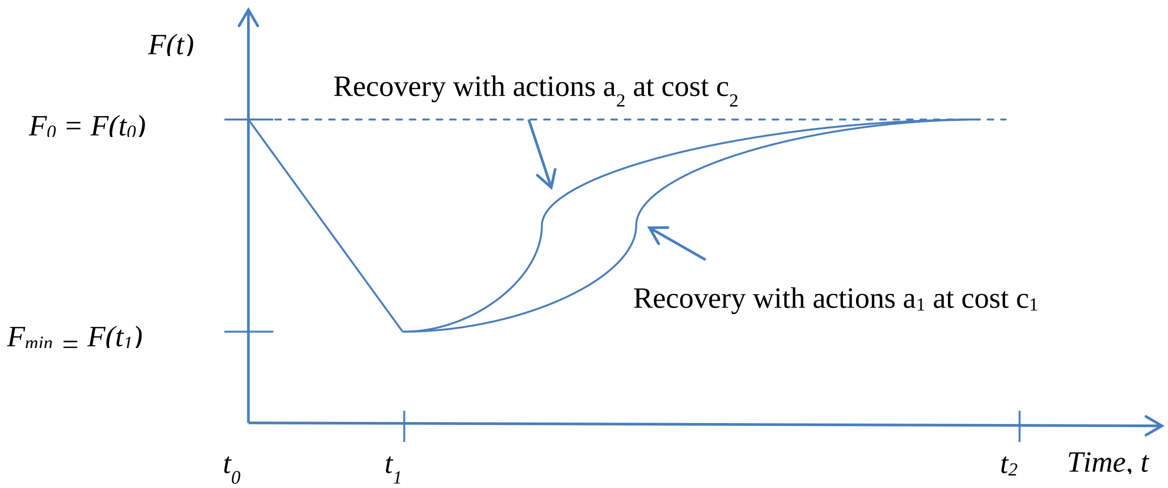

Figure 2 Recovery variation under different recovery strategies

In any particular application, SI itself may be composed of several different measures of system performance (with relative weights), and those weights together with the factor α serve to normalize Z to some useful scale. For a water distribution system, usually the impact of damage is the failure to satisfy water demand of the customers and hence can be a representation SI. The aim of the optimization model is to schedule repair tasks so as to minimize the TRE and SI.

In section below, we formulate an optimization model for damage recovery of water distribution systems.

2. Optimization Overview

Problems related to the selection of the best among many options are framed in a branch of mathematics known as mathematical programming or mathematical optimization problems. In the sections below, we briefly describe the general concepts of optimization and multi-level optimization, specifically bi-level optimization, with help from two simple examples.

2.1 General Optimization

Let us consider a simple function $f(x) = x^2 - 2x$ where $x \in [-10,10]$. Suppose our aim is to find the minimum value the function, $f(x)$ can take; we then have an optimization problem in hand. The objective function of this optimization problem is $f(x)$, and the decision variable is *x*. This optimization problem has some special features. The problem is a constrained optimization problem because *x* is constrained to take values only between and including -10 to 10 (called the feasible set or feasible space). In general, constraints can be in the form of functions also. An optimization problem is called a non-linear optimization problem if either the objective function or any constraint function is non-linear in terms of decision variable, *x*. Since the objective function, $f(x)$ of the present optimization problem is non-linear, it is a non-linear optimization problem. In specific terms, the present problem is a quadratic programming problem because the objective function is a quadratic function of *x* and the constraints $x \leq 10$ and $x \geq -10$ are linear in *x*.

The problem can be easily solved by plotting the value of $f(x)$ corresponding to different values *x*. The plot $f(x)$ for *x* between -10 and 10 is shown in Figure 3. It can be seen that the function $f(x)$ has minimum value when $x = 1$. The value of *x* for which objective function $f(x)$ attains minimum value is called an optimal solution; in this case $x = 1$ is the optimal solution. The minimum value of the function is equal to -1.

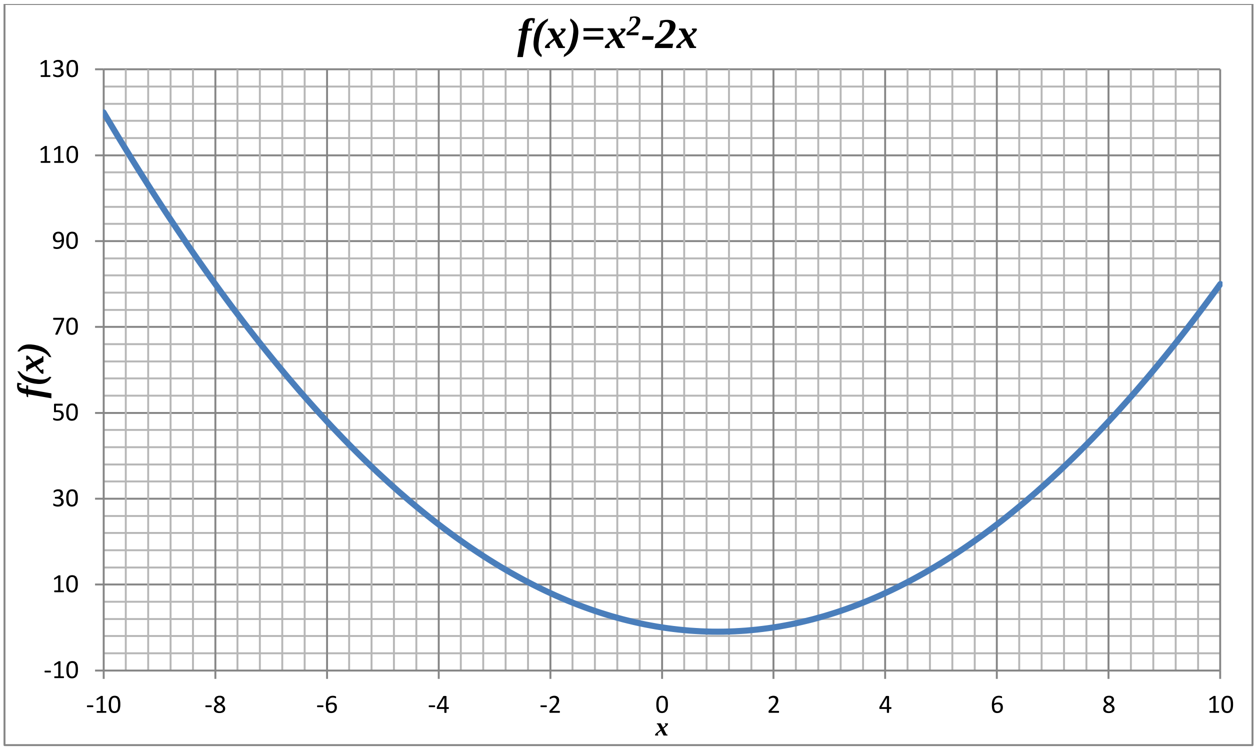

Figure 3 Optimization example

The problem can also be solved analytically. From basic calculus, a function has a minimum or maximum value at a point where the first derivative, $f'(x)$ of the function $f(x)$ is equal to zero. The point gives a minimum function value if the second derivative, $f''(x)$, is postive, and maximum if it is negative. If we obtain the first derivative of $f(x) = x^2 - 2x$ and equate it to zero to solve for *x,* we get:

$$f'(x) = 2x - 2 = 0$$
$$\Rightarrow x = 1$$

We now take the second derivative to check whether x=1 gives a mimum function value or maximum.

$$f''(x) = 2$$
$$\Rightarrow f''(x = 1) = 2$$

The second derivative is positive implying that the function value $f(x=1)=-1$, is the minmum value of this function. In fact, one generalization is that, for a single variable function, if the second derivative is positive everywhere in the feasible space, the optimization problem is called a convex problem; The optimal solution is global optimal– the point where the function has lowest value in the entire feasible space. We then observe an interesting point regarding the example above: even if we relax the assumption $x \in [-10,10]$ to $x \in \Re$, where $\Re$ denotes the set of real numbers, the optimal solution would be same, $x=1$!

Some difficulties arise when dealing with non-linear optimization problems. In general, non-linear funtions can have multiple points where the first derivative is zero and second derivative is postive, leading to multiple possible optimal solutions. These solutions are called local optima. Out of these multiple optima, only one solution is possibly the global optimal; finding that solution is a difficult endeavor especially when there are many variables involved. In many real world optimization problems, the underlying objective funtion or the constraints are non-smooth, in a sense that either the first derivative or second derivative does not exist at some points in the feasible region

The general form of optimization problems is as follows

$$\begin{aligned}&\min f(X)\\&s.t.\\&g_i(X) \le k_i\end{aligned}$$

where s.t., stands for subject to, X is a vector of decision variables, f is the objective function, g_i are the constraint functions, and k_i is a constant of ith constriant.

2.2 Bi-level Optimization

In this section, we briefly describe the bi-level optimization problem [*Aiyoshi*, 1981; *Jerome Bracken and McGill*, 1973; 1974; *J. Bracken and McGill*, 1978; *W. Candler and Norton*, 1977; *Wilfred Candler and*

Townsley, 1982; *Moore and Bard*, 1990], using a very simple example. Let us extend the optimization example in previous section by assuming the objective function includes two variables, *u* and *x*, and that *x* is itself a solution to an optimization problem that depends on *u*. Consider the optimization problem given as follows:

$$
\begin{aligned}
&\min\ f(u,x) = u + x \\
&\text{s. t.} \\
&u \in \{-10, 10\}
\end{aligned}
$$

and *x* is the solution to:

$$
\begin{aligned}
&\min_{x/u}\ \mathrm{g(x)} = -x \\
&\text{s.t.} \\
&x \in \{-10, 10\} \\
&x - 2u \le 0
\end{aligned}
$$

There are two optimization problems in hierarchy. Such problems are called bi-level optimization problems. The upper level problem is $f(u,x)$, and the lower level problem is $g(x)$. In a bi-level optimization problem, once a value of *u* is selected in the upper level, the value of *x* is choosen in the lower level given the selected value of *u*. Hence the choice of value *x* is affected by the choice of value of *u*. There is a complex interaction between the upper level problem and the lower level problem. The upper level decision, *u*, affects the lower level decision directly via the upper level objective function, and indirectly through the constraints in the lower level problem. The lower level decision, x, also affects the upper level problem in a similar way. We try to understand this interaction by solving the example problem. Considering only the variable *u*, the upper level objective function is minimum when $u = -10$. Selecting *u* = -10 as the upper level decision, the lower level needs to choose that value of *x* which minimizes $g(x)$. Due to *u* = -10, the constraint in the lower level problem becomes, $x \le -20$, which is outside the feasible region of *x*, [-10 10]. Because the lower level problem is rendered infeasible, the upper level decision has to be changed. Now, if $u = 10$, the constraint in the lower level problem becomes $x \le 20$, making the entire region [-10, 10] available for x. The optimal solution for the lower level problem now is x=10. The upper level function value is now equal to 20. This, however, may not be the minimum function value the upper level problem can attain. Selecting $u = -5$, we should have, $x \leq -10$ in the lower level problem. The there is only one feasible solution to the lower level problem, $x = -10$.The lower level objective value becomes 10, and the upper level objective value is -15. This is clearly a better overall solution, and in fact, this is the optimal solution for the bi-level problem. If u is increased further, we see that the upper level objective funtion increase correspondingly.

In the above problem, we tried to show how the lower level objective is influenced by the upper level objective function, and how the upper level objective function is affected by the lower level problem. When deciding in the upper level problem, there should be some feasible space available for the lower level problem to select its solution from.

Bi-level optimization problems are generally difficult to solve [*Hansen et al.*, 1992; *Jeroslow*, 1985] because of being non-convex [*Fortuny-Amat and McCarl*, 1981], having disconnected feasible solution space, piece wise objective function in the upper level optimization problem, and because of being non-differentiable [*Colson et al.*, 2007]. However, they provide a great scope for the cases where two or more decision makers, in heirarchy, make decisions independently, as opposite to a single decision maker making decision about many objectives, for example in multi-objective optimization.

3. Optimization Model Formulation

In this section, we formulate a bi-level optimization model for the recovery of a water distribution network after a damage. In the damaged state, it might not be possible to meet the required water demands of the entire water distribution network. However, some supply may be made to some sections of the system. For the purpose of analysis, water demand of different sections of water distribution system are usually aggregated as nodal demands in a network. Suppose some elements of a water distribution system, for example, pipes supplying water, pumps pumping water from a reservoir, and some tanks for water storage, are damaged. The objective is to repair the water distribution system in such a way that minimizes the total cost of the system. The total cost, Z, of the system is given as [*Turnquist and Vugrin*, 2013; *Vugrin et al.*, 2010].

$$Z = SI + \alpha TRE$$

where SI is System Impact, TRE is Total Recovery Cost, and α is as defined earlier. For the present study, we have used weighted unmet water demand as a measure of SI.

$$SI = \beta \sum_{n} \sum_{T} (D_{n,t} - d_{n,t}) s_n$$

$d_{n,t}$ = demand satisfied at node *n* in time period *t* in cubic meters. Here, we assume that Flow Control Valves (FCV's) are present and serviceable at all nodes.

$D_{n,t}$ = actual demand at node *n* in time period t in cubic meters.

s_n is the weighting factor assigning relative priority to nodes.

β is a factor for converting the demand to monetary value.

TRE can be calculated as the total cost of repair of all the damaged elements. For the repair of each element, certain tasks need to be performed. Some tasks can have different modes of execution; for example, a task can be performed in an expedited mode or in a normal mode. The expedited mode may allow quicker repair, but may require more resources and cost more than the normal mode. Let $C_{i,j,m}$ be the cost associated with task *j,* performed in mode *m,* on damaged element *i*. TRE is then calculated as follows:

$$TRE = \sum_{i} \sum_{j} \sum_{m} \sum_{t} C_{i,j,m} u_{i,j,m,t}$$

where,

$$u_{i,j,m,t} = \begin{cases} 1 & \text{if task } j \text{ performed on element } i \text{ is initiated in time period t, to be executed in mode } m \\ 0 & \text{Otherwise} \end{cases}$$

The complete bi-level optimization model formulation for the water distribution network recovery is given as:

$$\min\ Z = SI + \alpha TRE = \sum_{n}\sum_{T}(D_{n,t} - d_{n,t})s_n + \alpha\sum_{i}\sum_{j}\sum_{m}\sum_{t} C_{i,j,m}u_{i,j,m,t} \tag{1}$$

The decision variable for the upper level problem, recovery optimization, is $u_{i,j,m,t}$. The constraints for this optimization problem are described below:

1. Tasks require k types of resources, in certain quantity, $r^{k}_{i,j,m}$, in each time period during which they are active. At each time period there are limited resources available of each resource type. Let $R_{k,t}$ be the available resource of type k at time period t. The resource used of type k by all active tasks at each time period t should not exceed the available resources.

$$\sum_{i}\sum_{j}\sum_{m}\sum_{\tau=t-\eta_{i,j,m}+1}^{t} r^{r}_{i,j,m}u_{i,j,m,\tau} \leq R_{k,t} \quad \forall k,t \tag{2}$$

Here, $\eta_{i,j,m}$ is the time required to complete task j on element i in mode m.

2. Task j for element i should be executed only in one mode, and should not be scheduled more than once. Some tasks may not be scheduled at all.

$$\sum_{m}\sum_{t} u_{i,j,m,t} \leq 1\ \forall i,j \tag{3}$$

3. Tasks should be perfomed in a particular order depending on precedence requirements provided by task networks. The precedence requirement constrains the initiation of task l before the completion of another task, say task j. The below constraint manifests that task l can not be started before the completion of task j (both related to damaged element i in the system.

$$\sum_{t} t \sum_{m} u_{i,l,m,t} \geq \sum_{t} \sum_{m} (t + \eta_{i,j,m}) u_{i,j,m,t} \tag{4}$$

4. The repair of each damaged element, i, helps to satisfy some additional water demand. The repair of damaged elements is indicated by the completion of specific tasks, j'. After a damaged element is repaired, the set of undamaged elements L^c is augumented by the repaired element, i. If $L^c(0)$ is the set of undamaged elements at time period zero, we determine the set of undamaged elements at the end of time period, t as follows.

$$L^c(t) = L^c(0) \bigcup \left[\bigcup_{i} \left\{ i \sum_{\tau}^{t-\eta_{i,j',m}} u_{i,j',m} \right\} \right] \tag{5}$$

When a specific task, j', of element i, is completed, element i is added to the undamaged element set L^c.

5. Since, at each time period, it's required to minimize the unmet demand, we need to add the (lower level) optimization problem of minimizing unmet demand to the (upper level) recovery optimization problem. The lower level optimization and its constraints become constraints of the upper level optimization problem, Equation 1.

$$\min_{n,t} \sum_{n} \sum_{T} (D_{n,t} - d_{n,t}) s_n \tag{6}$$

Subjected to:

a. Continuity equations at demand nodes: For all demand nodes, inflow into a node is equal to the outflow from the node plus the demand met. $q_{n,n',t}$, denotes that flow leaves node *n* and enters node *n'* at time *t*, and $q_{n',n}$ denotes flow leaving node *n'* and entering node *n* at time *t*.

$$-\sum_{n'} q_{n,n',t} + \sum_{n'} q_{n',n,t} = d_{n,t} \quad \forall n,t \tag{7}$$

b. Source node constraints: Outflow from the source node, *w,* is less than its capacity, $S_{w.t}$ and no flow into the source node at any time.

$$\sum_{n'} q_{w,n',t} \le S_{w,t} \quad \forall w,t \tag{8}$$

$$q_{j,w,t} = 0 \ \forall w,t \tag{9}$$

c. At all times, demand satisfied at each node should not be more than the required demand.

$$d_{n,t} \le D_{n,t} \forall n,t \tag{10}$$

d. The water distribution elements have limited capacity. Specifically, we considered flow capacity of pipes and pumps in this study. Flow occurring through these elements should not exceed their flow capacity. The flow in these elements can be in only one direction within any time period.

$$q_{n,n',t} \le Q_{n,n'} Z_{n,n',t} \forall (n,n')\{(n,n'),(n'n)\} \in L^c \tag{11}$$

$$Z_{n,n',t} = \begin{matrix} 1 & \text{flow occurring in arc } (n,n')\ \{(n,n'),(n'n)\} \in L^c \\ 0 & \text{flow not occurring in arc } (n,n')\{(n,n'),(n'n)\} \in L^c \end{matrix}$$

$Q_{n,n'}$, is the flow capacity of arc with flow going from node *n* to node *n'*. Two arcs or two flow directions, (n,n') and (n',n), exist for an element. For an element to be working, both flow directions need to be possible, and if an arc is in the damaged set L, no flow is possible.

$$Z_{n,n',t} + Z_{n',n,t} \le 0\ \forall \{(n,n'),(n',n)\} \in \mathrm{L} \tag{12}$$

Flow occurs only in one direction, either from node *n* to *n'* or from node *n'* to *n*, not both, in undamaged element set, L^c

$$Z_{n,n',t} + Z_{n',n,t} \le 1\ \forall \{(n,n'),(n',n)\} \in \mathrm{L}^c \tag{13}$$

e. Energetic constraints: The law of conservation of energy should be followed while flow occurs between nodes. Head loss between two nodes, *n* and *n'*, is equal to the energy consumed by flow and by friction. This constraint is written in the form of empirical equation provided by Hazen-Williams [*Gardner*, 1933] and is used for calculating the head loss between two nodes.

$$(h_n - h_{n'} = K_{n,n'} q^{\alpha}{}_{n,n'}) \langle 1 \rangle_{q_{n,n'} \ne 0}\ \forall (n,n')\{(n,n'),(n',n)\} \in \mathrm{L}^c \tag{14}$$

h_n and $h_{n'}$ are the heads at nodes, *n* and *n'* respectively. *K* and α depend on pipe characteristics and are considered to be known. In this study, we have used $\alpha = 1.85$, and *K* is calculated as given by the Hazen-William's Equation.

$$K = \frac{10.67L}{C^{1.852}(2r)^{4.87}} \tag{15}$$

Where, C is pipe roughness coefficient depending on pipe charectaristics, L is the length of pipe in meters and r is radius of pipe in meters.

$\langle 1 \rangle_{q_{n,n'} \neq 0}$ is an indicator funtion, indicating that the constraint is invoked only when there is some flow in the pipe. For elements with pumps, the constraint is modified to include the extra energy provided by the pump for the flow. The equation is given below. Here, we have used Hazen-William's equation for calculating head loss or energy loss.

$$h_w - h_{n'} = k_{w,n'} q^{\alpha}{}_{w,n'} - p(a_1 + a_2 q_{w,n'} + a_3 q_{w,n'}^2) \forall (w,n') \in \mathrm{L}^c \tag{16}$$

Here, head at the source, h_w is equal to elevation, e_w at the reservoir or river. $a_1, a_2,$ and a_3 are the constants given by pump specifications. *p* = 0 or 1, if pump is not working, and working respectively.

f. Finally, the system should guarantee flow with a minimum pressure at each node, hence hydraulic head at node *n* should be at least the minimum head required plus the elevation of the node.

$$h_n \geq h_{\min} + e_n \tag{17}$$

This completes the model formulation for recovery of water distribution system. The model described is very general; depending on the requirements, the objective funtion can be manipulated and constraints can be added. As can be perceived, the optimization model formulated is quite complex. Bi-level optimization problems, as mentioned earlier, are in general non-convex and non-smooth. The feasible region for the lower level problem may be disconnected. Non-linearity due to the energetic constraint in the lower level problem adds to the complexity. For solving the optimization problem, we used a heurustic optimization technique called simulated annealing. The sub-section below briefly explains the idea of simulated anealing.

3.1 Simulated Annealing

Simulated Annealing (SA) is a heuristic optimization method for complex optimization problems. Simulated Annealing idea is based on a paper by [*Metropolis et al.*, 1953] and was proposed for optimization by [*Kirkpatrick et al.*, 1983]. The idea is closely analogous with annealing in thermodynamics. In annealing, a metal is heated to a certain temperature and then cooled under control so that the size of crystals produced is large and defects are minumum. These properties of the metal depend on the rate of cooling. If the rate of cooling is slow, large crystals will be formed; however, if it's fast, crystals may contain imperfections.

In Simulated Annealing, a feasible solution is randomly generated, and the objective value is calculated. The solution is either accepted or rejected depending on some probability distribution. The probability of accepting a worse solution (in terms of objective function value) is slowly decreased as an analogy to the slow decrease in temperature in annealing. In this way, better solutions are found upon iterations. A solution is accepted as an optimal solution when a better solution cannot be found after a specified number of iterations. Solutions that do not improve the objective are accepted, although with small probabilities, because it prevents the algorithm to get trapped in local solutions.

For the present study, simulated annealing is used to generate solutions for the upper level problem, i.e. to generate repair schedules, and the lower level problem is solved using Excel's non-linear Generalized Reduced Gradient (GRG) solver. The scheduling of repair tasks in the present study provides an example of the Multi-mode Resource Constrained Project Scheduling Problem(MRCPSP) [*Chiang et al.*, 2008; *Damak et al.*, 2009; *Jarboui et al.*, 2008; *Lin-Yu and Shih-Chieh*, 2009; *Sprecher and Drexl*, 1998].

3.2 Simulated Annealing Model for Multi-Mode Project Scheduling

Several authors have used simulated annealing to address the MRCPSP (see for example, [*Boctor*, 1996; *Bouleimen and Lecocq*, 2003; *Józefowska et al.*, 2001]). There are particularly intersting ideas in Boctor's approach that lend themselves well to recovery optimization problem. A core idea of Boctor's approach to the project scheduling problem is that a potential solution can be described as an ordered list, or a *sequence,* of tasks. The sequence implies a schedule, which can be constructed relatively easily. In the multi-mode case, the sequence also contains mode selection for each task (which implies its duration and resource requirements). Sequences can also be checked easily for validity (i.e., no task can appear in the sequence before any of its required predecessors, or after any of its successors).

To evaluate a sequence as a possible solution, the schedule of task starting times (and modes, when applicable) must be determined in a way that respects the resource and precedence constraints. This schedule then implies the times for milestones at which partial or full capacity restoration occurs on individual links.

To make idea more clear, consider project network in Figure 4. The restoration of full capacity of damaged system element occurs at the milestone indicated as F (completion of all 1 through 8 tasks), and at the milestone represented by node C in the network, 40% of capacity is restored.

The tasks require varying amounts of two resources, each of which is available in limited quantity. For the first 10 periods after the disruption, 4 units of each resource are available. After the tenth period, additional resources can be acquired and the availability increases to 6 units of each resource.

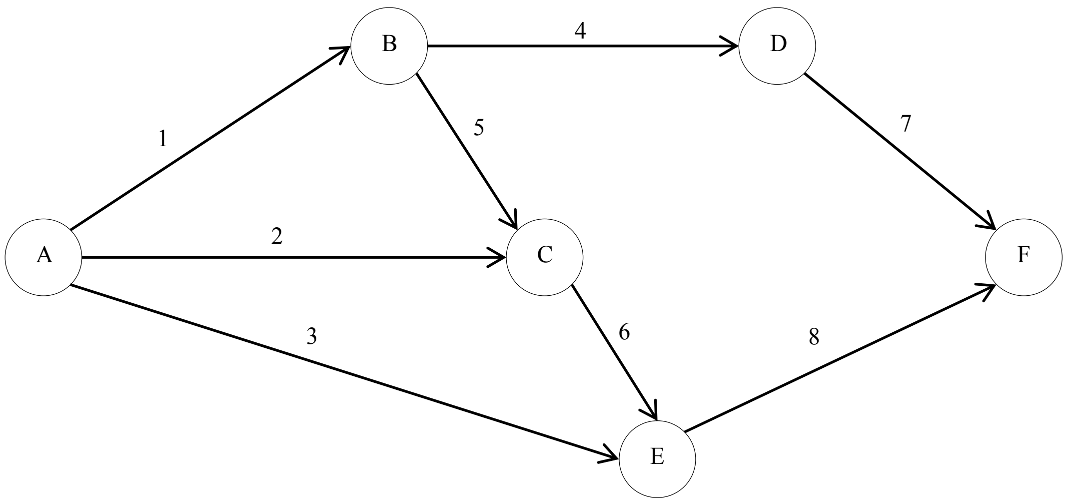

Figure 4 Example of restoration project and precedence

Table 1 indicates the task characteristics for the project. Tasks 5 and 6 can be done in one of the two possible modes, with different durations, resource requirements and costs. For later notational convenience, each of the link-task-mode (*ijm*) combinations in Table 1 has been assigned an index.

Table 1 Recovery project characteristics for example

Index	Task Number	Mode	Duration (Periods)	Cost	Resource Requirements (Units)	
					Resource 1	Resource 2
1	1	1	4	80	2	0
2	2	1	4	320	2	3
3	3	1	6	300	1	2
4	4	1	3	90	1	1
5	5	1	4	160	2	1

6	5	2	2	180	3	2
7	6	1	7	420	2	2
8	6	2	4	480	4	4
9	7	1	4	200	3	1
10	8	1	3	30	1	0

To make a schedule for recovery, we first make sequences. The task sequences:

[1 2 3 4 5 6 7 8]   [2 1 5 3 6 8 4 7]   [1 4 5 2 6 3 7 8]

are all valid sequences for this network because each task always appears after all of its predecessors and before any of its successors. However, the sequence:

[1 5 2 4 3 8 6 7]

is invalid because task 8 appears before one of its predessors, task 6.

To convert a sequence to a schedule, each task is examined in order and scheduled to begin at the earliest time at which its predecessors are complete and there are sufficient resources available. For example, using the task durations and resource requirement in Table 1 and assuming four units of both resources are available in each period, the first valid sequence listed above can be scheduled using the following steps:

1. Task 1 is scheduled to start at time 0.
2. Task 2 is also scheduled to begin at time 0, using the remaining two units of resource 1 and three units of resource 2.
3. Task 3 is scheduled to begin at time 4, when task 2 ends, freeing the necessasry resources.
4. Task 4 is scheduled to begin at time 4, when its predecessor, task 1, finishes.

5. Task 5 has two possible modes. For the moment, let us assume that mode 1 is chosen. The task is then scheduled to begin at time 4, when task 1 (its predecessor) finishes and task 2 also finishes, freeing the necessary resources.
6. Task 6 also has two modes, and let us assume for the moment that for it also mode 1 is chosen. Then it is scheduled to begin at time 8, when its predecessor, task 5, finishes.
7. Task 7 is scheduled to begin at time 15, when task 6 finishes and the required units of resource 1 become available.
8. Task 8 is scheduled to begin at time 15, when its predecessor, task 6, finishes.

The completion of the effort is at time 19, when task 7 finishes. The milestone at node C, implying partial restoration of capacity, occurs at time 8 when task 2 and 5 are both complete.

For both tasks 5 and 6, we arbitrarily chose mode 1 for their execution in constructing this schedule. For evaluation of specified sequence, we also want to try other choices for the modes on these two tasks. In this simple example, only two tasks have multiple modes, and only two choices for each task, so there are only 4 combinations to be evaluated and it is straightforward to evaluate all of them. In larger problems, where there are many more possible combinations of mode choices for tasks, a subset of different combinations can be chosen at random, with the best result being used to characterize the sequence.

Once a sequence is evaluated, the idea of simulated annealing is to indetify the neighboring solution that may be an improvement. In this application, a neighboring solution is constructed by shifting the position of one task in the sequence to another valid position. Thus, the sequence [1 2 3 5 4 6 7 8] would be considered a neighbor of [1 2 3 4 5 6 7 8] because task 5 has been moved ahead of task 4. The shift to create a neighbor is *not* a swap of the positions of two tasks (although in the example cited here the effect is same). We pick a task and shift its position in the list, with the tasks between original position and its new position sliding up or down as necessary.

The simulated annealing process selects a neighbor of a current solution by chosing a task in the sequence randomly and moving it to a randomly chosen valid position in the list, adjusting the elements of the sequence as necessary. This new sequence is then evaluated by creating a schedule for the tasks and then doing network flow calculations at the time points corresponding to milestones. Whether the neighboring point is accepted as preferable to the original point or not depends on the parameters of the simulated annealing process. The search continues until no further improvements can be made.

Two important implementation elements improve the efficiency of the search considerably. The first is that the solution process can memorize the network flow values for specific capacity conditions. In the small example above, the only capacity conditions that matter are zero capacity, 40% capacity, and full capacity. Different sequences produce the capacity changes at different times, but this does not affect the evaluation of network flow. Thus, for this example, once three network flow compuations have been done, no more are neccesary for SI evaluations of different sequences. The SI computations are related to when the shifts in capacity occur, but the calculation is very simple.

In more complicated networks and more complex disruptions, many more network flow camputations are generally necessary, but the idea of memorizing the flow calculations under specific capacity conditions still reduces the computational burden very substantially. This general idea has been noted by Bocchini and Fragopol (2012) in their algorithm. They use a genetic algorithm, rather than simulated annealing, and are solving a somewhat different optimization, but they observe that in later stages of their search process approximately two-thirds of the required solutions were simply retrieved from previously stored results and did not have to be recomputed.

A second important computational element for the simulated annealing process is that because capacity changes on links occur at discrete times, it is quite easy to characterize solutions that are dominated. That is, if the order of link capacity restorations is the same in potential solution *a* as it is in potential solution *b*, but in solution *a* none of the changes occur later than in *b*, and at least one change occurs earlier, we know that solution *a* will dominate solution *b* in the calculation of SI. In this case, if we already have computed SI for solution *a*, we need not bother with the calculations for solution *b*. We are better off discarding it and going on to some other candidate solution. Both the memorizing of previous calculations and exploiting dominance are effective in reducing the computational burden of the optimization process.

We apply the optimization model to a small water distribution system in section 4 and to a much larger one in section 5.

4. Model Application 1

In this section, we apply our optimzation model to a small water distribution network obtained from [*Halhal et al.*, 1997]. In section 4.1 below we describe the network. In section 4.2 we set the stage for model application, including discussion of the damage scenario and recovery requirements. In section 4.3, we present and anlayse the optimal schedule produced by the recovery optimization model.

4.1 Network

The network is shown in Figure 5. The network consists of 15 pipes, 9 demand nodes, and a reservior. The network is fairly small and is not very complex as it does not have any pumps or tanks. However, it has good nodal connectivity. Some demand can be satisfied via undamaged pipes, and as the damaged pipes get repaired, additional demand can be satisfied. Different competitive recovery schedules are expected to have different relative effect on the upper level and lower level objective function. The pipe data for the network is given in Table 2 and node data is provided in Table 3. From Table 3, we see that total demand is .245m^3/s.

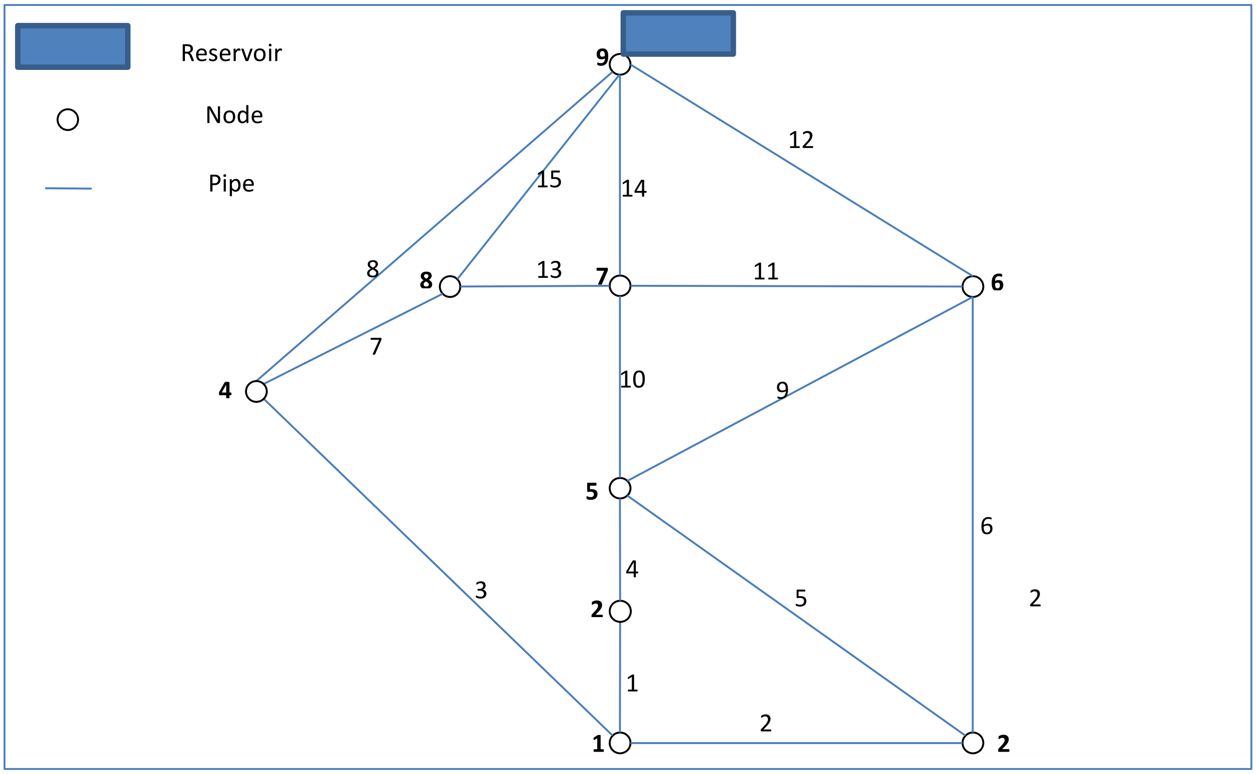

Figure 5 Water Distribution Network Layout

Numbers near circles are node IDs. Numbers alongside lines represent Pipe IDs. We will use a short form to denote pipes and nodes. Pipes will be denoted as PID; for example, Pipe 2 will be denoted by P2. Similarly nodes will be denoted by NID; for example Node 2 will be denoted by N2.

Table 2 Pipe Data

Pipe ID P_	Length(m)	Diameter (m)	Hazen William's Coefficient C
1	1300	0.08	130
2	1500	0.08	130
3	1500	0.08	130
4	1700	0.15	70
5	3100	0.2	100
6	1200	0.15	120
7	1000	0.3	110
8	2100	0.15	100
9	1500	0.15	120
10	3600	0.2	80

11	2000	0.3	100
12	3000	0.2	110
13	1000	0.2	120
14	1500	0.15	90
15	1000	0.2	90

Table 3 Node Data

Node ID N_	Average Daily demand (m^3/s)	Elevation(m)
1	0.015	20
2	0.015	22
3	0.03	20
4	0.07	16
5	0.02	33
6	0.055	30
7	0.025	21
8	0.015	22
9	Reservoir	200

4.2 Setup for Model Application

We created a hypothetical damage scenario in which three elements of the water distribution system are damaged: P8, P14 and P15. With these pipes out of service, the network is connected to the source reservior N9 only through P12, which has limited capacity.

For the repair, we created a hypothetical repair task precedence network for the project, as shown in Figure 6. Each row corresponds to repair tasks of a Pipe. There are 18 repair tasks, including three dummies denoting the end repair of each element, required for the complete recovery of the system. Four resources are required for repair tasks: Resource1 (Inspector units), Resource 2 (Engineering units), Resource 3 (Special labor units) and Resource 4 (General Labor units). At each time period, there are 2 Inspector units , 4 Engineering units, 5 Special Labor units, and 10 General Labor units available.

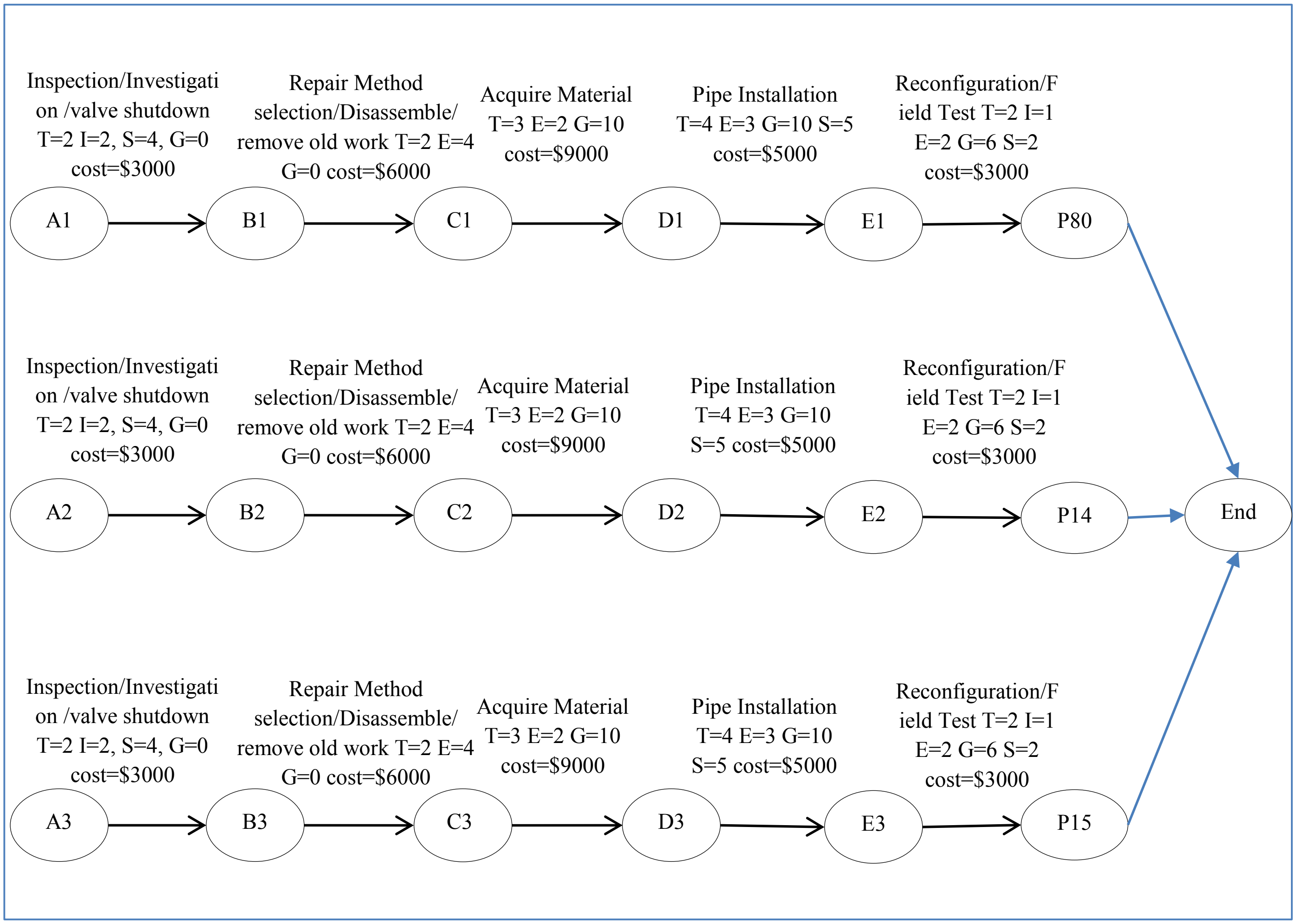

Figure 6 Task Precedence Network

Table 4 below describes the task charactaristics.

Table 4 Characteristics of Recovery Tasks

Task ID	From Node	To Node	Duration (4-hr)	Resource 1	Resource 2	Resource 3	Resource 4	Cost (*$1000)
1	A1	B1	2	2	0	4	0	3000
2	B1	C1	2	0	4	0	0	6000
3	C1	D1	3	0	2	0	10	9000
4	D1	E1	4	0	3	5	10	5000
5	E1	P8	2	1	2	2	6	3000
6	A2	B2	2	2	0	4	0	3000
7	B2	C2	2	0	4	0	0	6000
8	C2	D2	3	0	2	0	10	9000

9	D2	E2	4	0	3	5	10	5000
10	E2	P14	2	1	2	2	6	3000
11	A3	B3	2	2	0	4	0	3000
12	B3	C3	2	0	4	0	0	6000
13	C3	D3	3	0	2	0	10	9000
14	D3	E3	4	0	3	5	10	5000
15	E3	P15	2	1	2	2	6	3000
16	P8	End	0	0	0	0	0	0
17	P14	End	0	0	0	0	0	0
18	P15	End	0	0	0	0	0	0

4.3 Schedule Optimization

The optimal schedule of all the tasks required for the recovery of the water distibution system is show in Figure 7. In this example, all tasks are required for the recovery of system, and each task can be done in only one mode, so TRE is constant. Hence, the optimal schedule in Figure 7 is result of minmizing the SI on the system.

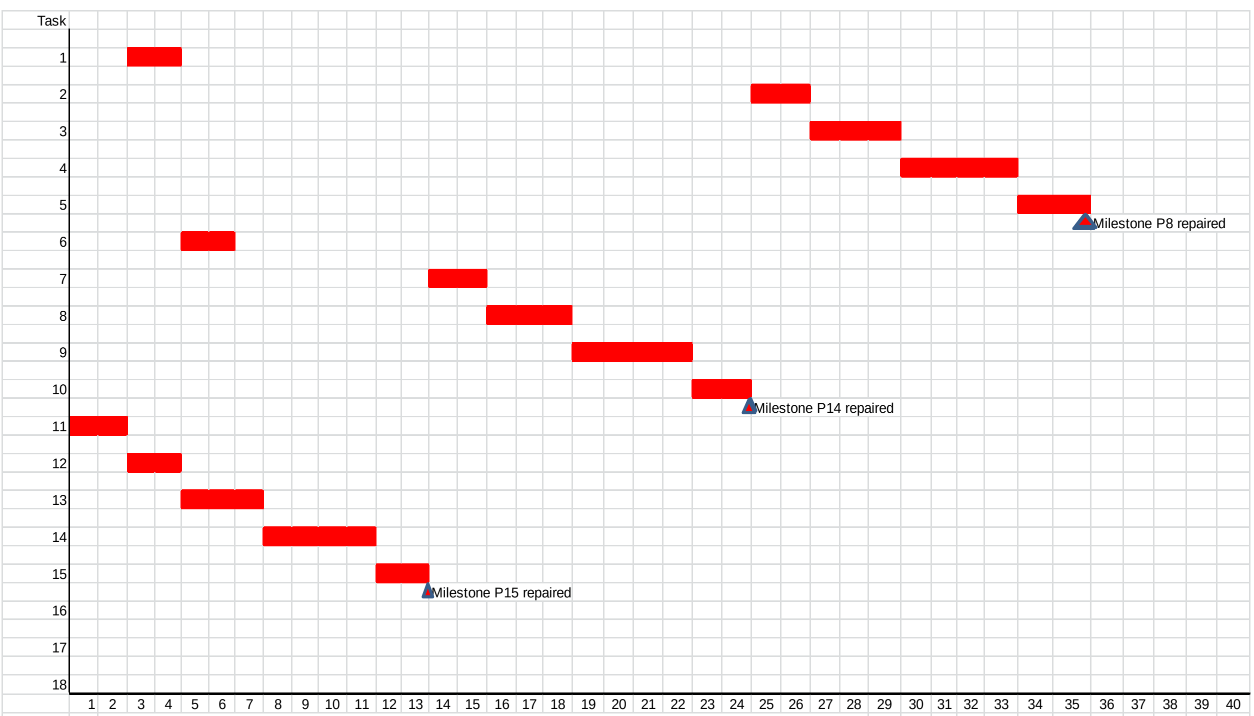

Figure 7 Optimal Schedule of Repair Tasks

Repair of P15 starts at the first time period, when task 11, the first repair task of P15, is initiated. During time periods 1 and 2, task 11 uses all the available inspectors, preventing the initiation of any repair task of P14 or P8. Figure 8 shows the usage of Inspectors (Resource 1). The subsequent figures, Figure 9, Figure 10, and Figure 11 show the usage of Engineers, Special Labor and General Labor, respectively. In time periods 3 and 4, P15 repair continues with task 12 being performed, and task 1 of P8 is also started. During these time periods, we might actually like to initaite repair task of P14 rather than P8, but as can be seen in repair schedule, Figure 7, working on task 1 of P8 does not actually delay the repair of P14, because the second task of either P8 or P14 can not started until P15 is completely repaired. At time period 5, task 6 of P14 is initiated.

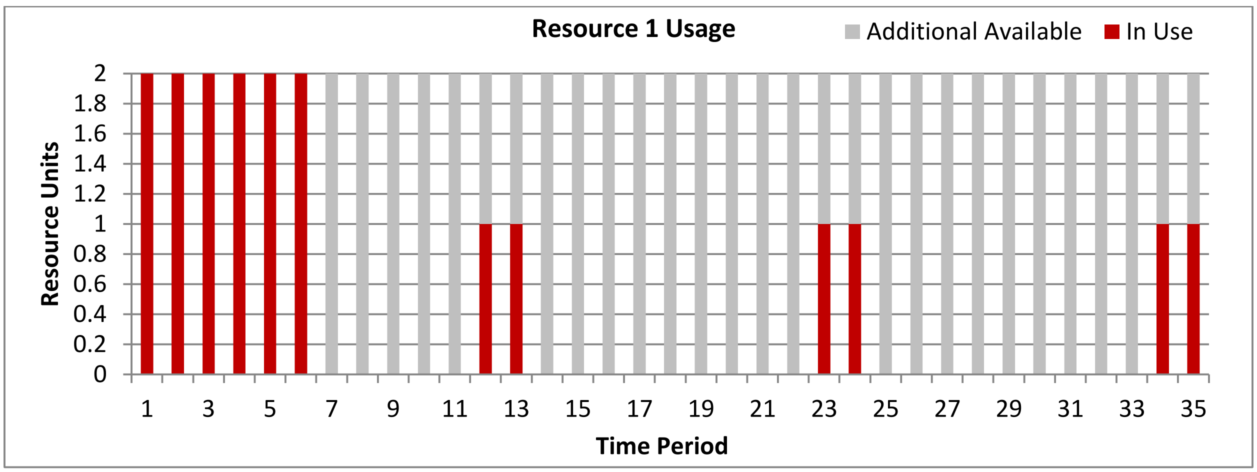

Figure 8 Inspector (Resource 1) Usage

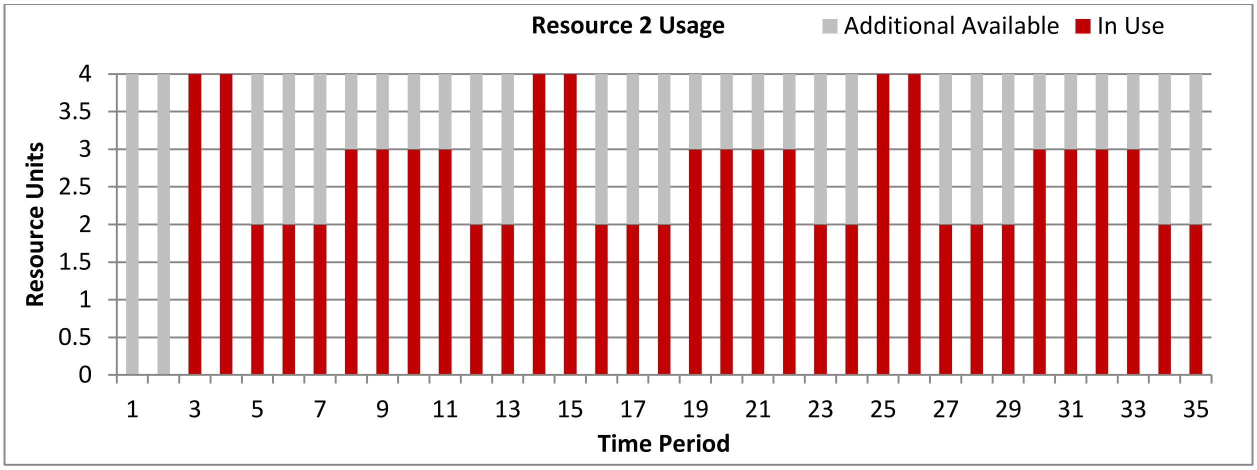

Figure 9 Engineer (Resource 2) Usage

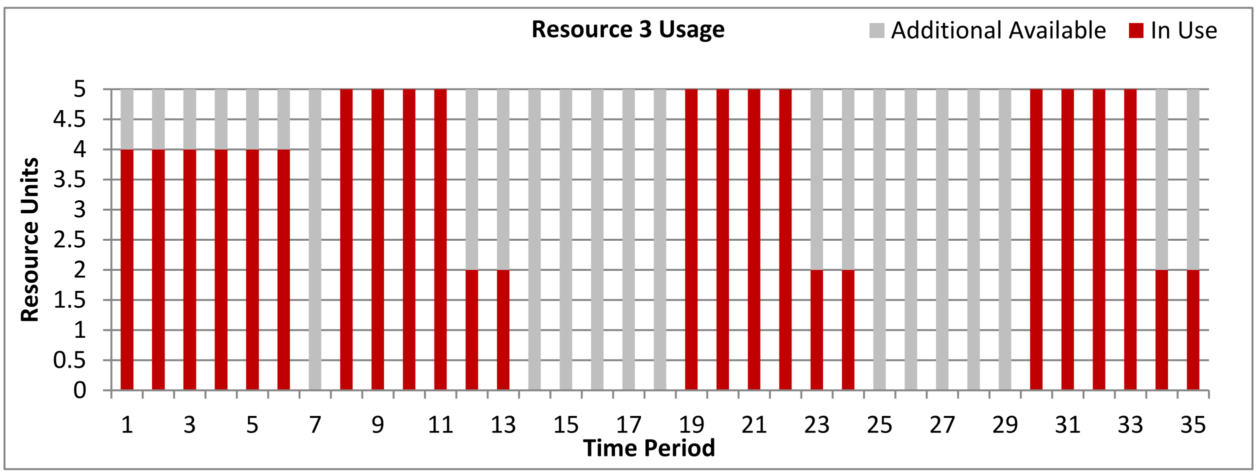

Figure 10 Special Labor (Resource 3) Usage

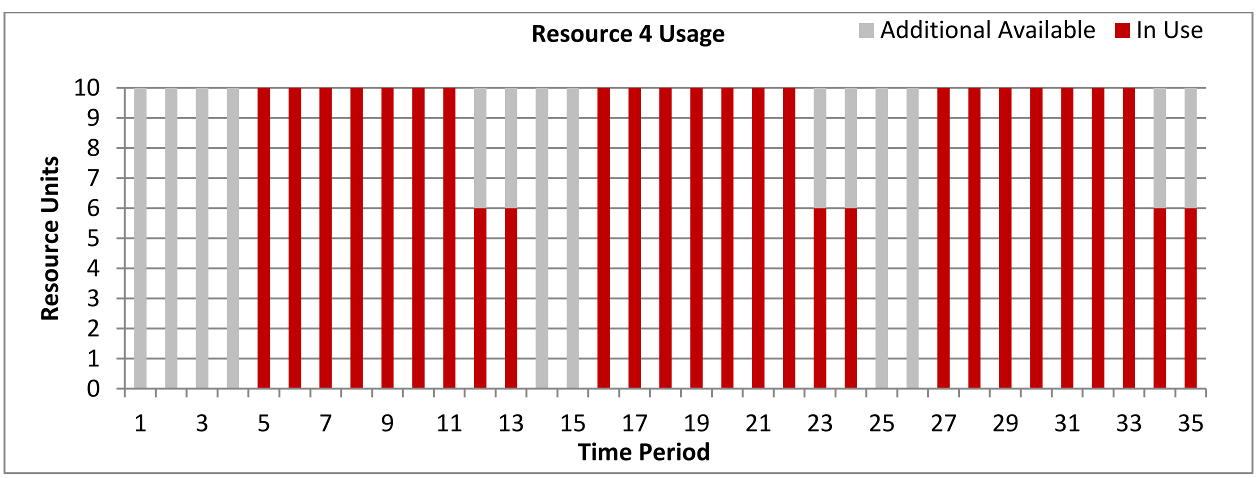

Figure 11 General Labor (Resource 4) Usage

During time periods 5 to 11, all our available General Labor units are being used for repair of P15 as can be seen in Figure 11. The two tasks that are feasible in time periods 7 to 13 are task 2 and task 7 of P8 and P14, respectively. Both these tasks require 4 engineering units; however, there are fewer than 4 units available during these time periods as can be seen in Figure 9. At end of time period 13, P15 repair is completed, and the cost per hour due to unmet demand, H, calculated using equation 18 is reduced to \$6350 from \$38000 dollars, as can be seen in Figure 12, which shows the system impact on the system. At time period 14, task 2 and task 7 of P8 and P14, are both feasible; task 7 is choosen because of P14 earlier repair being more important than repair of P8. Repair of P14 continues till the end of time period 24 with out any hinderence. Fewer engineers being available than required for task 2 during time periods 14 to 24, repair of P8 is delayed. At time period 24, P14 is repaired, and H reduces to \$532. In time period 25, repair of P8 is started again, and goes on till the end of time period 35 when the repair is complete. The entire water distribution system is recovered, and brought back to its original form.

The equation for calculating the cost due to unmet demand, H, is given below.

$$H = \frac{\beta \sum_{n} (D_n - d_n) s_n}{4*60*60} \tag{18}$$

D_n and d_n are actual demand and satisfied demand in a 4-hour period. β is a conversion factor for converting cubic meter to dollars, here taken equal to 1000. Node weights, S_n are assumed to be 1 for all nodes. Using Table 3, we can calulate $\sum_{n} D_n = .245*60*60*4 = 3528m^3$. Eventually, H gives us the cost due to unmet demand per hour.

The Systemic Impact (SI) on the water distribution system is shown in Figure 12. SI is calculated as the aggregate of cost incurred per hour due to unmet demand, H, over the entire duration of recovery project. The cost per hour due to unmet demand,H, at the time of damage is \$38000 thousands of dollars. This condition continues until the end of period 13 when Pipe P15 is repaired. After that, additional water can be supplied through P15, reducing the unmet demand and hence H, till the end of period 24. At the end of period 24, the unmet demand is reduced further. The complete recovery of the system occures at the end of time period 35. After period 35, there is no unmet demand, hence H is zero. SI over all 35 periods of complete recovery of the system is equal to \$2.281 million. The TRE for the recovery of the system is \$780000.

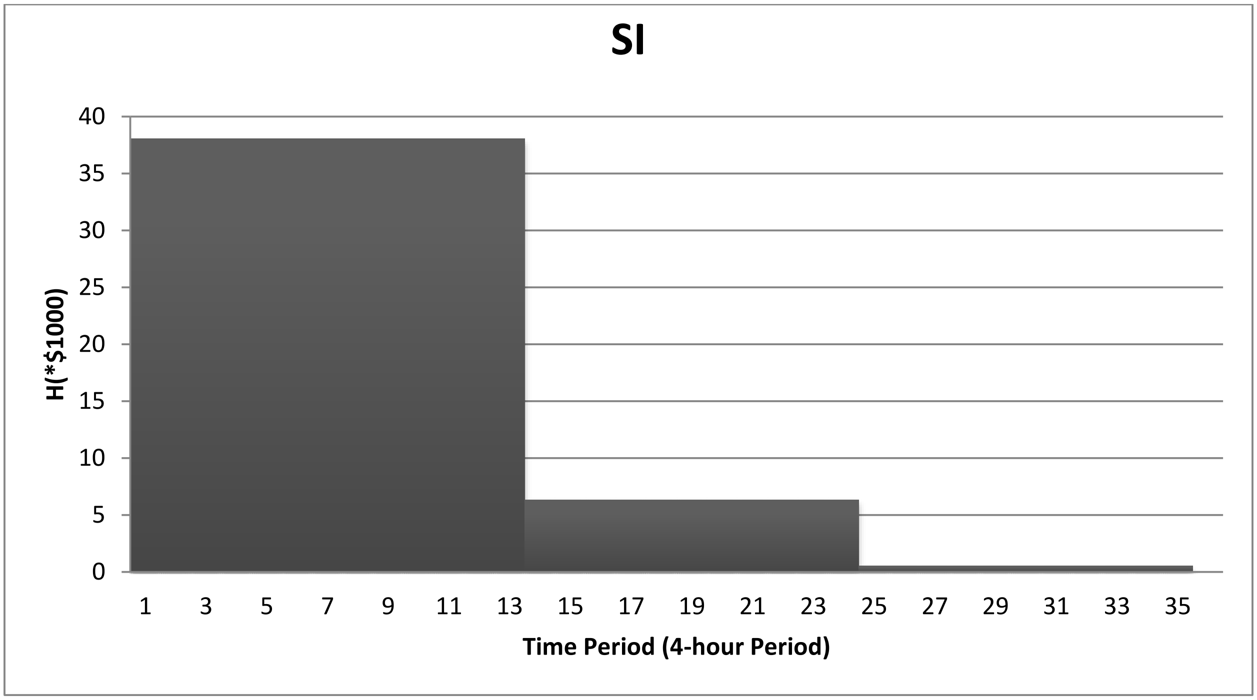

Figure 12 Optimal Systemic Impact (SI) on Water Distribution System

The optimal recovery of the three damaged elements, P8, P14 and P15, is performed in the sequence [P15, P14, P8]. To see if this is actually the best repair sequence, we calculated the demand satisfaction of all other possible repair sequences. There are a total of 6 repair sequences possible for 3 damaged elements. All the repair sequences and their respective demand satisfied as each element repairs is shown in Table 5. For example, the first sequence listed is [P8, P14, P15]. When no element is repaired yet (just at the time of disruption), the demand satisfied in the system is $0.092 m^3/s$. We see when P8 is repaired demand satisfied becomes .141m^3/s; after that when P14 is repaired, demand satisfied reaches to .190m^3/s and at last when P15 gets repaired, the networks is back in original undamaged form and all demand (.245m^3/s) is satisfied.

Table 5 Damage Repair Sequences

Sequence 1	None	P8	P14	P15
Demand Satisfied(m^3/s)	0.093	0.141	0.190	0.245
Sequence 2	None	P8	P15	P14
Demand Satisfied(m^3/s)	0.093	0.141	0.239	0.245
Sequence 3	None	P14	P8	P15
Demand Satisfied(m^3/s)	0.093	0.140	0.190	0.245
Sequence 4	None	P14	P15	P8
Demand Satisfied(m^3/s)	0.093	0.140	0.242	0.245
Sequence 5	None	P15	P8	P14
Demand Satisfied(m^3/s)	0.093	0.219	0.239	0.245
Demand Satisfied(m^3/s)	None	P15	P14	P8
Demand Satisfied	0.093	0.219	0.242	0.245

From the table above, clearly out of all the six repair sequences, sequence [P15, P14, P8] is best in terms of demand satisfaction after each element is repaired. The optimization model schedules the repair tasks to attain this sequence. This validates the recovery model.

The next section deals with a larger and more complex water distribution system.

5. Model Application 2

In this section, we apply our optimization model to a larger and more complex water distribution system called the Anytown network. We describe the network in Section 5.1below. We set the stage for model application for three different experiments in Section 5.2, and finally, we provide and analyze the optimal schedules of three different experiments.

5.1 Anytown Network

Anytown Network [*Walski et al.*, 1987] is a water distribution system of a hypothetical small city. The distribution system is depicted in Figure 13. The water source for Anytown network is a nearby river, Node 10 in Figure 13. There is no other source of water available. Three pumps of the same technical specifications draw water from the river to supply to the town. The pumps work in parallel for supplying higher flow rate at a given pressure head. The tanks receive water during non-peak demand hours and deliver during peak demand hours, thus preventing head deficiency and making efficient use of energy by running the pumps at lesser energy levels than what would be required otherwise. The original aim of framing the network by [*Walski et al.*, 1987], was to come up with different methods of optimal design of new elements, like pipes, pumps and tanks, and choose pipes that need renovation or redesign so as to cope with anticipated developments in coming years. In this regard, Anytown network is studied extensively (see for example, [*Farmani et al.*, 2005; *Murphy*, 1994; *Ralph*, 2009; *Walters et al.*, 1999]). This study adopts the design and data from [*Walters et al.*, 1999]. Anytown network is a complex network and has all the typical elements of a realistic water distribution system. The network has tanks, pumps, complex combinations of 41 pipes, and the topography is not very even. The data for the network is given in Table 6 and Table 7. It is straightforward to calculate the total demand of the system equaling $0.618 m^3/s$.

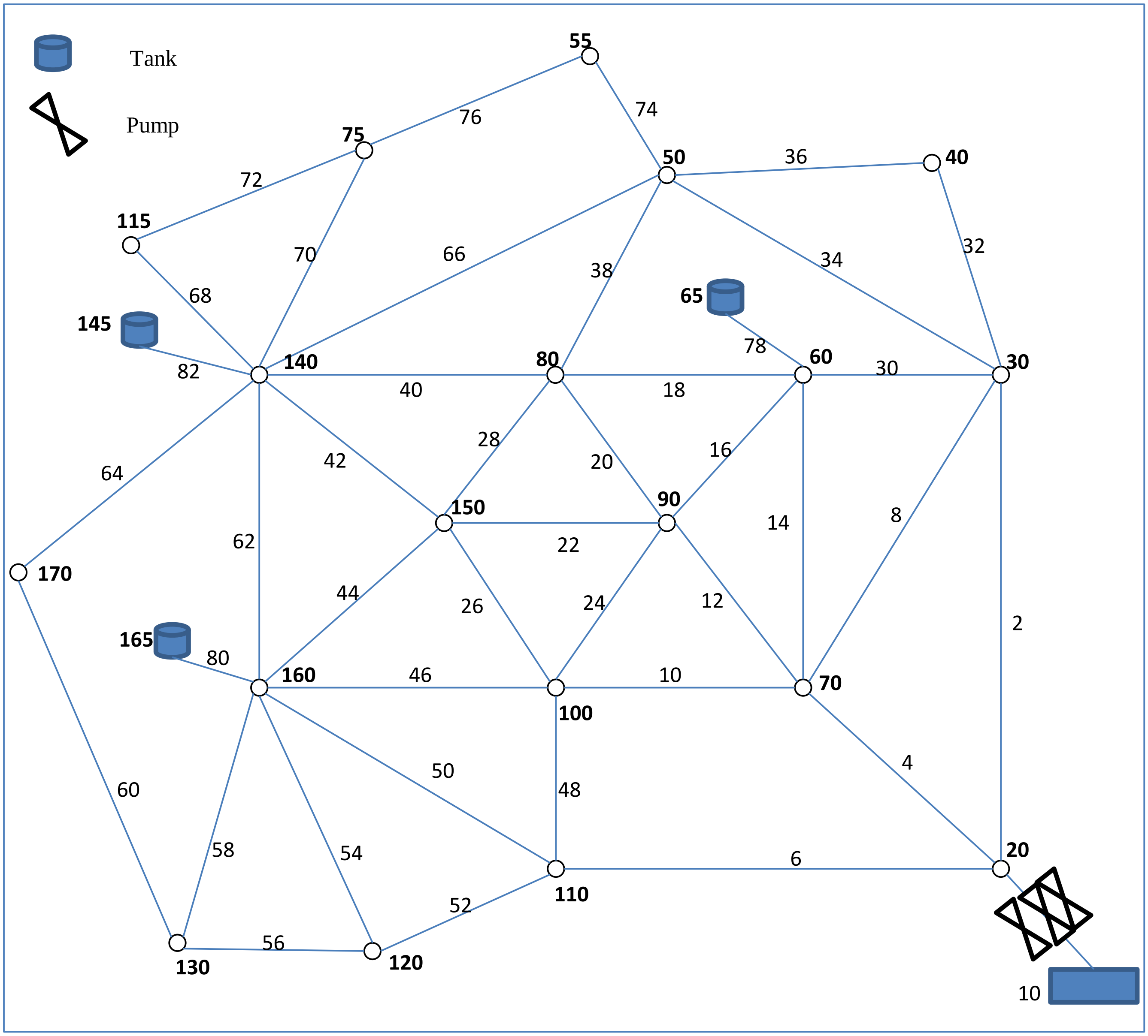

Figure 13 Anytown Network Layout

Numbers near circles are node ID. Numbers alongside lines represent Pipe ID. Numbers near tanks represent tank ID. Number alongside reservoir is its ID. Tank 165 will be denoted by P80 henceforth.

Table 6 Anytown Pipe data

Pipe ID_	Length(m)	Diameter (meters)	Hazen William's Coefficient	Pipe ID	Length(m)	Diameter (meters)	Hazen William's Coefficient
2	3657.6	0.4064	70	64	3657.6	0.2032	120
4	3657.6	0.3048	120	66	3657.6	0.2032	120
6	3657.6	0.9144	70	68	1828.8	0.3048	130
8	2743.2	0.3048	70	70	1828.8	0.3048	130
10	1828.8	0.3048	70	72	1828.8	0.1524	130
12	1828.8	0.254	70	74	1828.8	0.254	130
14	1828.8	0.3048	70	76	1828.8	0.254	130
16	1828.8	0.254	70	78	30.48	0.8128	120
18	1828.8	0.3048	70	80	30.48	0.508	120
20	1828.8	0.254	70	82	30.48	0.508	130
22	1828.8	0.254	70				
24	1828.8	0.254	70				
26	1828.8	0.3048	70				
28	1828.8	0.254	70				
30	1828.8	0.254	120				
32	1828.8	0.254	120				
34	2743.2	0.254	120				
36	1828.8	0.254	120				
38	1828.8	0.254	120				
40	1828.8	0.6096	120				
42	1828.8	0.2032	120				
44	1828.8	0.508	120				
46	1828.8	0.2032	120				
48	1828.8	0.2032	70				
50	1828.8	0.8636	120				
52	1828.8	0.4572	120				
54	2743.2	0.1524	130				
56	1828.8	0.2032	120				
58	1828.8	0.5588	120				
60	1828.8	0.508	120				
62	1828.8	0.7112	120				

Table 7 Anytown Node data

Node ID N_	Expected Daily Demand (m^3/s)	Elevation (meter)
10	River	3.048
20	0.031545	6.096
30	0.012618	15.24
40	0.012618	15.24
50	0.037854	15.24
55	0.037854	24.384
60	0.031545	15.24
65	Tank	65.532
70	0.031545	15.24
75	0.037854	24.384
80	0.031545	15.24
90	0.06309	15.24
100	0.031545	15.24
110	0.031545	15.24
115	0.037854	24.384
120	0.025236	36.576
130	0.025236	36.576
140	0.025236	24.384
145	Tank	64.008
150	0.025236	36.576
160	0.06309	36.576
165	Tank	65.532
170	0.025236	36.576

5.2 Setup for Model Application

In this analysis, a constant daily demand, shown in Table 7, equal to average daily demand during year 2005, is assumed. A second assumption is that, at the time of damage, both tanks, N65 and N165, are full. A third assumption is that no tank gets filled until all the elements are repaired and the system is completely recovered. Together, the three assumptions are quite reasonable for the present study. Tanks only get filled during non-peak hours when excess pump capacity is available. Given the assumed average daily demand, it is unlikely that any tank would receive water while the network is still in damaged state. The effect of the assumption of tanks being full at the time of damage is that the maximum amount of demand is satisfied. The damage scenario created here is quite severe. Two out of three pumps (Pump1 and Pump2), plus pipes P2, P4, P6, P48, P50 and P80 are out of service. In this damage scenario, all demand except that at node N20, is entirely cut off from the water source. Until some element is repaired, we can satisfy only N20 demand and the quantity available in undamaged tanks. The network, however, has quite high connectivity. The recovery scheduling problem becomes more difficult and interesting for a highly complex and highly reliable damaged network. For the recovery of the system, elements damaged require planned repair or replacement in order to get back to their original serviceable condition. The repair or replacement of each element requires different tasks that need to be completed in a specific order, provided in the form of a task precedence network. The task precedence networks developed hypothetically for the present study, for pipes, pumps and tank are shown in Figure 14, Figure 15, and Figure 16, respectively. All the pipes have the same task precedence network, as do the two pumps. Each task incurs some cost and requires certain supply of different resources. There are 56 tasks to be performed including 9 dummy tasks that do not require any resources, and do not have any cost associated. There are four types of resources required as in earlier case.

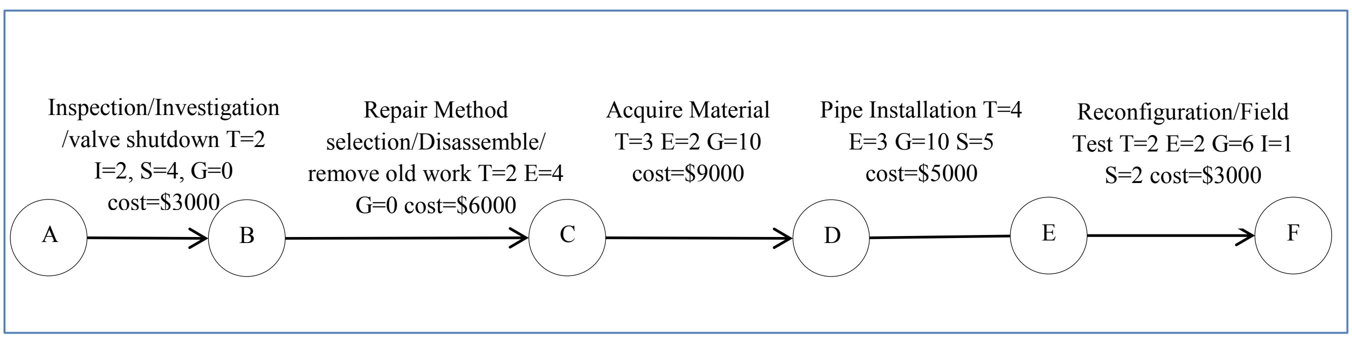

Figure 14 Task precedence for Pipes

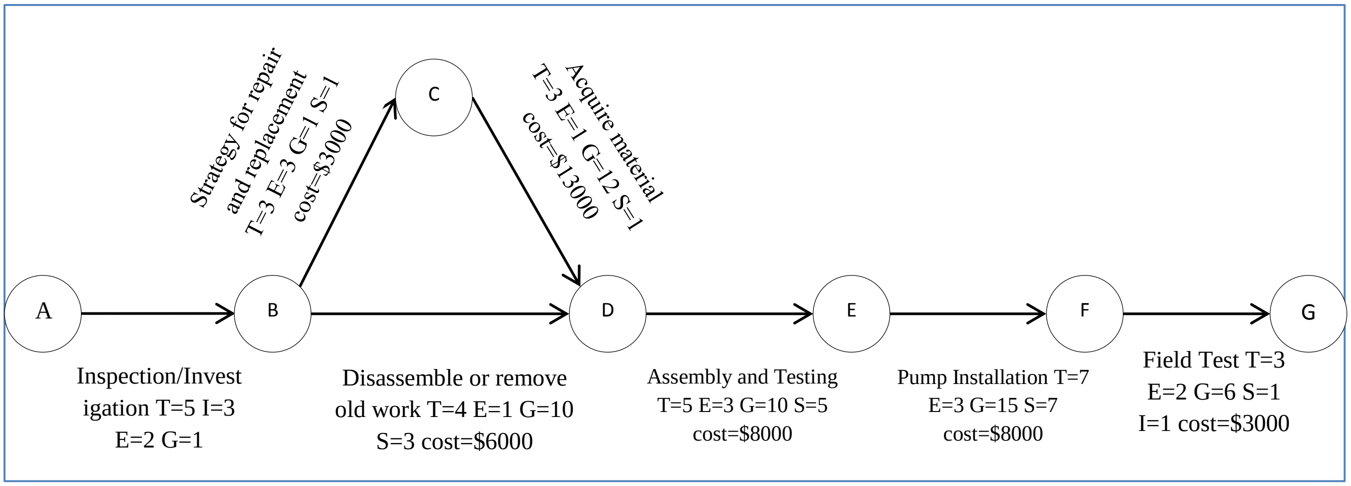

Figure 15 Task precedence for Pumps

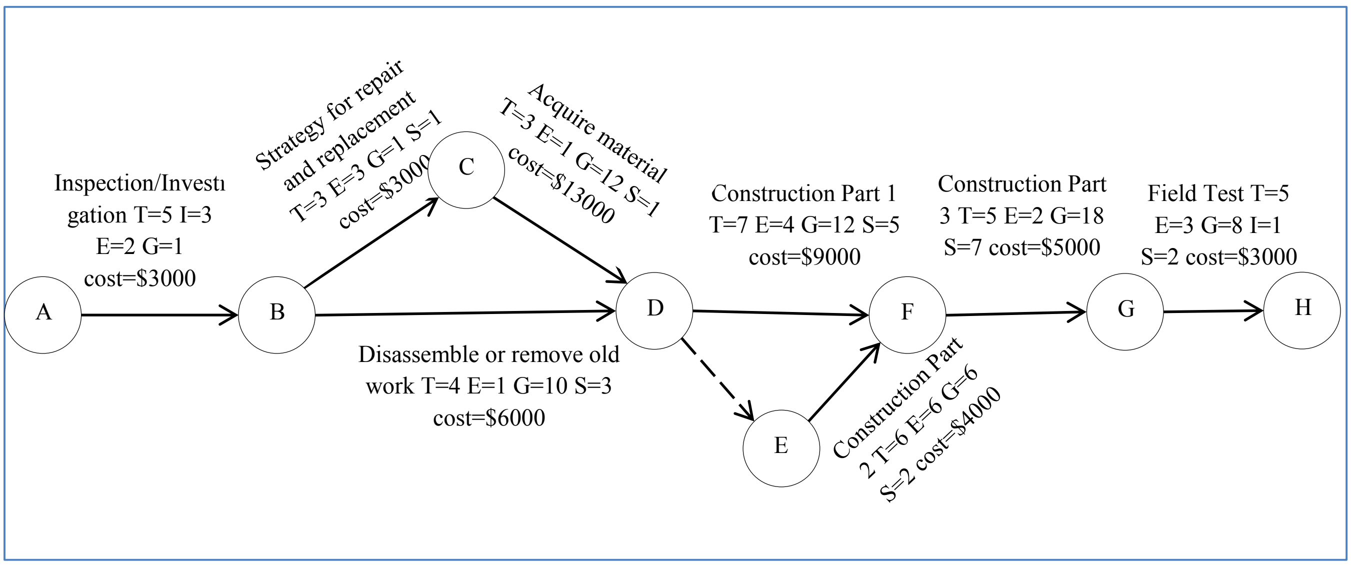

Figure 16 Task precedence for Tank

T= Time in 4hr, E= Engineering units, I= Inspector units, S= Special labor units, G= General labor units

5.3 Analysis of Optimal Schedules

Three experiments with the optimization model representing different scenarios of resource availability are performed. In the first experiment, resources available, of each resources type, at each time period, are just sufficient to meet the needs of the single task that requires the largest amount of it. This is the smallest amount of each resource that will guarantee that all tasks can be scheduled at some time. Table 8 below shows the resource requirements for all tasks to be completed for recovery of the Anytown water distribution system. For the first experiment, 3 Inspector units (Resource 1), 6 Engineering units (Resource 2), 7 Special labor units (Resource 3), and 18 units of General labor units (Resource 4), are available at each time period.

Table 8 Characteristics of Recovery Tasks

Task ID	From Node	To Node	Duration (4-hr)	Resource 1	Resource 2	Resource 3	Resource 4	Cost (*$1000)
1	A1	B1	5	3	2	0	1	3000
2	B1	C1	3	0	3	1	1	3000
3	B1	D1	4	0	1	3	10	6000
4	C1	D1	3	0	1	1	12	13000
5	D1	E1	0	0	0	0	0	0
6	D1	F1	7	0	4	5	12	9000
7	E1	F1	6	0	6	2	6	4000
8	F1	G1	5	0	2	7	18	5000
9	G1	P80	5	1	3	2	8	3000
10	A2	B2	5	3	2	0	1	3000
11	B2	C2	3	0	3	1	1	3000
12	B2	D2	4	0	1	3	10	6000
13	C2	D2	3	0	1	1	12	13000
14	D2	E2	5	0	3	5	10	8000
15	E2	F2	7	0	3	7	15	8000
16	F2	PPUMP1	3	1	2	1	6	3000
17	A3	B3	5	3	2	0	1	3000
18	B3	C3	3	0	3	1	1	3000
19	B3	D3	4	0	1	3	10	6000
20	C3	D3	3	0	1	1	12	13000
21	D3	E3	5	0	3	5	10	8000
22	E3	F3	7	0	3	7	15	8000
23	F3	PPUMP2	3	1	2	1	6	3000
24	A4	B4	2	2	0	4	0	3000
25	B4	C4	2	0	4	0	0	6000
26	C4	D4	3	0	2	0	10	9000
27	D4	E4	3	0	3	5	10	5000
28	E4	P2	2	1	2	2	6	3000
29	A5	B5	2	2	0	4	0	3000
30	B5	C5	2	0	4	0	0	6000
31	C5	D5	3	0	2	0	10	9000
32	D5	E5	3	0	3	5	10	5000
33	E5	P4	2	1	2	2	6	3000

34	A6	B6	2	2	0	4	0	3000
35	B6	C6	2	0	4	0	0	6000
36	C6	D6	3	0	2	0	10	9000
37	D6	E6	3	0	3	5	10	5000
38	E6	P6	2	1	2	2	6	3000
39	A7	B7	2	2	0	4	0	3000
40	B7	C7	2	0	4	0	0	6000
41	C7	D7	3	0	2	0	10	9000
42	D7	E7	3	0	3	5	10	5000
43	E7	P48	2	1	2	2	6	3000
44	A8	B8	2	2	0	4	0	3000
45	B8	C8	2	0	4	0	0	6000
46	C8	D8	3	0	2	0	10	9000
47	D8	E8	3	0	3	5	10	5000
48	E8	P50	2	1	2	2	6	3000
49	P80	End	0	0	0	0	0	0
50	PPUMP1	End	0	0	0	0	0	0
51	PPUMP2	End	0	0	0	0	0	0
52	P2	End	0	0	0	0	0	0
53	P4	End	0	0	0	0	0	0
54	P6	End	0	0	0	0	0	0
55	P48	End	0	0	0	0	0	0
56	P50	End	0	0	0	0	0	0

In the second experiment, called “Increased available resources”, the availability of each type of resource is increased by 50 percent, and rounded up to the nearest integer value. For example, available units of Special Labor would now be 1.5 times 7, rounded up to nearest integer, 11.

In the third experiment, named as “Multi-Mode Tasks”, some tasks have multiple modes of execution. These modes differ in the resources requirement, duration and costs. This allows the policy makers to decide whether to complete the tasks in expedited mode, requiring less time but probably more resources and cost, or in normal mode, which takes longer duration but requires fewer resources and costs less. The multi-mode task precedence networks for Pipes and Pumps are given in Figure 17 and Figure 18, respectively.

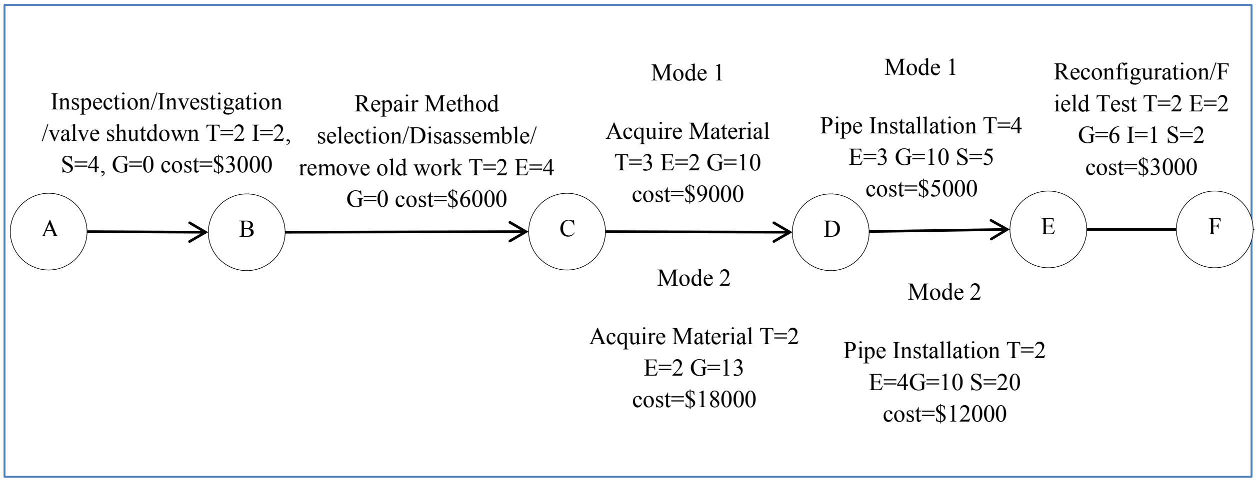

Figure 17 Multi-mode task precedence network of Pipes

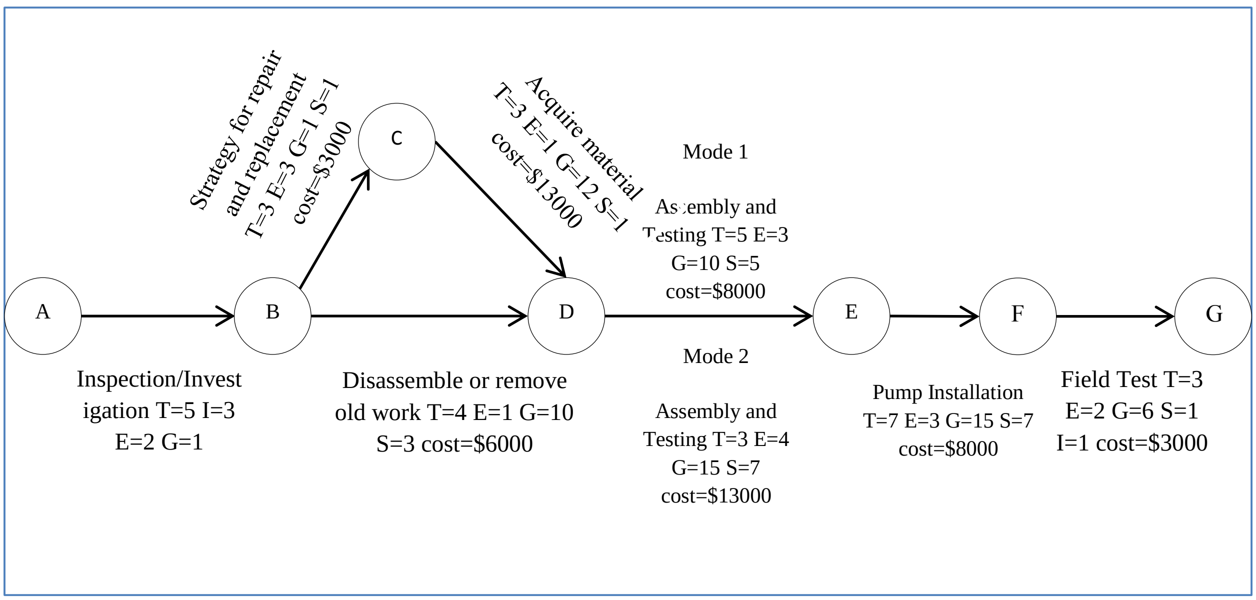

Figure 18 Multi-mode task precedence network of Pumps

The task characteristics of the multi-mode experiment are shown in Table 9. In this table, tasks 15, 17, 24, 26, 31, 33, 43, 45, 55 and 57 represent the additional modes available.

Table 9 Task Characteristics of Multi-Mode experiment

Task ID	From Node	To Node	Duration (4-hr)	Resource 1	Resource 2	Resource 3	Resource 4	Cost (*$1000)
1	A1	B1	5	3	2	0	1	3000
2	B1	C1	3	0	3	1	1	3000
3	B1	D1	4	0	1	3	10	6000

4	C1	D1	3	0	1	1	12	13000
5	D1	E1	0	0	0	0	0	0
6	D1	F1	7	0	4	5	12	9000
7	E1	F1	6	0	6	2	6	4000
8	F1	G1	5	0	2	7	18	5000
9	G1	P80	5	1	3	2	8	3000
10	A2	B2	5	3	2	0	1	3000
11	B2	C2	3	0	3	1	1	3000
12	B2	D2	4	0	1	3	10	6000
13	C2	D2	3	0	1	1	12	13000
14	D2	E2	5	0	3	5	10	8000
15	D2	E2	3	0	4	7	15	13000
16	E2	F2	7	0	3	7	15	8000
17	E2	F2	4	0	4	10	20	12000
18	F2	PPUMP1	3	1	2	1	6	3000
19	A3	B3	5	3	2	0	1	3000
20	B3	C3	3	0	3	1	1	3000
21	B3	D3	4	0	1	3	10	6000
22	C3	D3	3	0	1	1	12	13000
23	D3	E3	5	0	3	5	10	8000
24	D3	E3	3	0	4	7	15	13000
25	E3	F3	7	0	3	7	15	8000
26	E3	F3	4	0	4	10	20	12000
27	F3	PPUMP2	3	1	2	1	6	3000
28	A4	B4	2	2	0	4	0	3000
29	B4	C4	2	0	4	0	0	6000
30	C4	D4	3	0	2	0	10	9000
31	C4	D4	2	0	2	0	13	18000
32	D4	E4	3	0	3	5	10	5000
33	D4	E4	2	0	4	10	20	12000
34	E4	P2	2	1	2	2	6	3000
35	A5	B5	2	2	0	4	0	3000
36	B5	C5	2	0	4	0	0	6000
37	C5	D5	3	0	2	0	10	9000
38	D5	E5	3	0	3	5	10	5000
39	E5	P4	2	1	2	2	6	3000
40	A6	B6	2	2	0	4	0	3000
41	B6	C6	2	0	4	0	0	6000
42	C6	D6	3	0	2	0	10	9000
43	C6	D6	2	0	2	0	13	18000
44	D6	E6	3	0	3	5	10	5000

45	D6	E6	2	0	4	10	20	12000
46	E6	P6	2	1	2	2	6	3000
47	A7	B7	2	2	0	4	0	3000
48	B7	C7	2	0	4	0	0	6000
49	C7	D7	3	0	2	0	10	9000
50	D7	E7	3	0	3	5	10	5000
51	E7	P48	2	1	2	2	6	3000
52	A8	B8	2	2	0	4	0	3000
53	B8	C8	2	0	4	0	0	6000
54	C8	D8	3	0	2	0	10	9000
55	C8	D8	2	0	2	0	13	18000
56	D8	E8	3	0	3	5	10	5000
57	D8	E8	2	0	4	10	20	12000
58	E8	P50	2	1	2	2	6	3000
59	P80	End	0	0	0	0	0	0
60	PPUMP1	End	0	0	0	0	0	0
61	PPUMP2	End	0	0	0	0	0	0
62	P2	End	0	0	0	0	0	0
63	P4	End	0	0	0	0	0	0
64	P6	End	0	0	0	0	0	0
65	P48	End	0	0	0	0	0	0
66	P50	End	0	0	0	0	0	0

In this study, a task started cannot be stopped or paused until the task is completed. The optimal schedule of the above three experiments is analyzed in the following sections.

5.3.1 Minimum necessary resources

The optimal schedule provided by the optimization model is shown in Figure 19. Task 34 (corresponding to the repair of P6) is scheduled at the beginning of time period 1. Since we have limited inspector units, we cannot work any other precedence feasible repair task. Figure 20 shows the usage of Resource 1 (inspector units). Subsequent figures, Figure 21, Figure 22, and Figure 23 show the usage of Resource 2, Resource 3, and Resource 4, respectively. At time period 3, inspector units are available to start work on task 44 of P50. Limited inspector units availability at this period prevents the work on any other precedence feasible task.

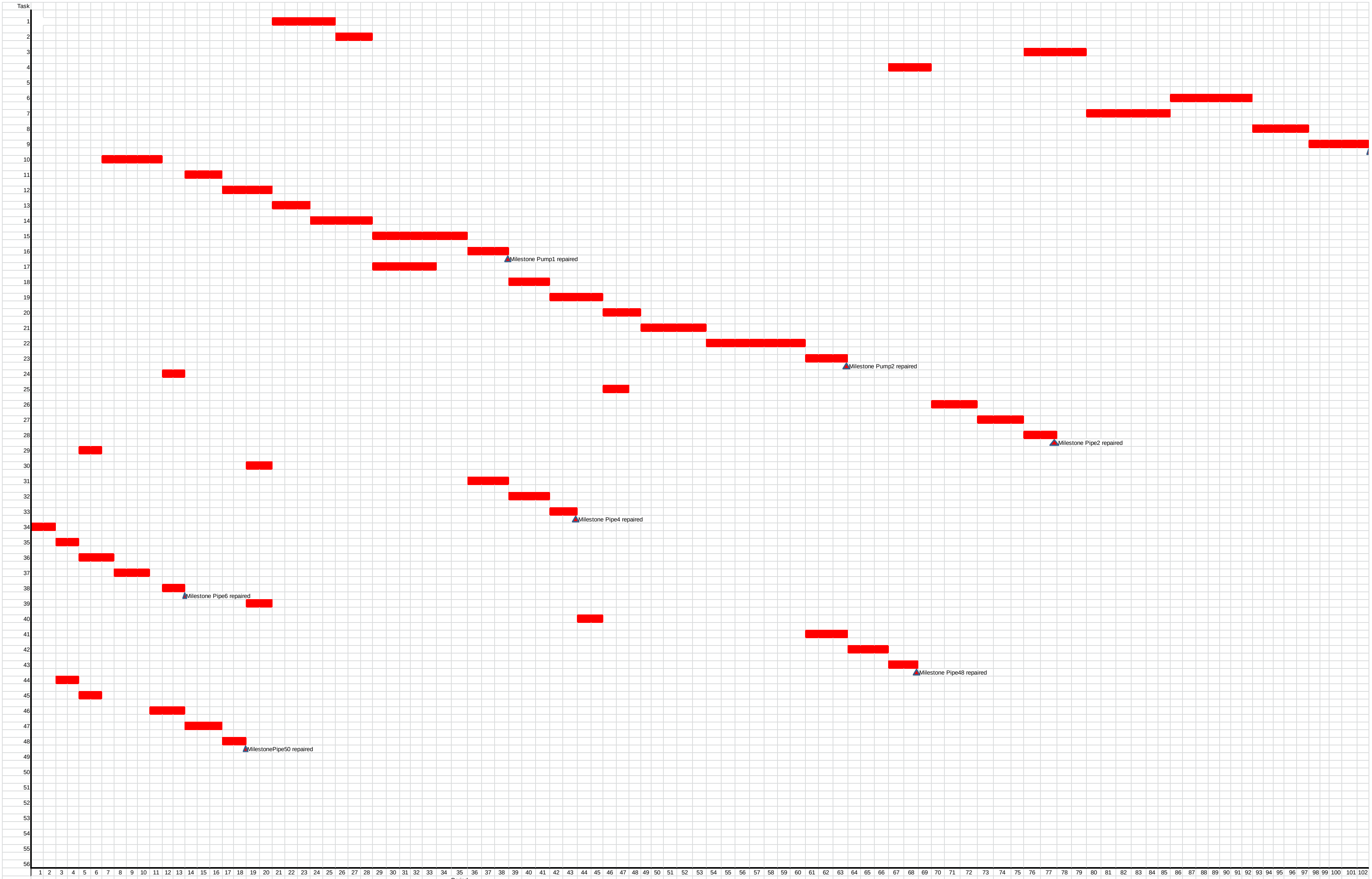

Figure 19 Optimal Schedule for first experiment

In time periods 5 and 6, while repair of P6 and P50 continues with tasks 36 and 45 being done, all the engineering units are being employed; see Figure 21. Engineering units unavailability prevents starting repair of Pump1, but resources are available to complete task 29, the first task of P4. In time periods 7 to 10, task 36 and task 37 of P6, utilize 10 of our available 18 general labor units. During this time period, because of less general labor units being available than required, task 46 of P50, requiring 10 general labors units, cannot take place. However, during this time period, resources are available to start work on Pump1. In time period 11, task 10 of Pump1 uses all the available inspector units, preventing task 38 from being done. At this time period, work on P50 is continued. In time periods 12 and 13, tasks 38 and 46 of P6 and P50, utilize all of 6 available engineering units, hence, tasks 11 and 30 of Pump1 and P4, cannot take place. Having enough resources available, we start task 24 of P4.

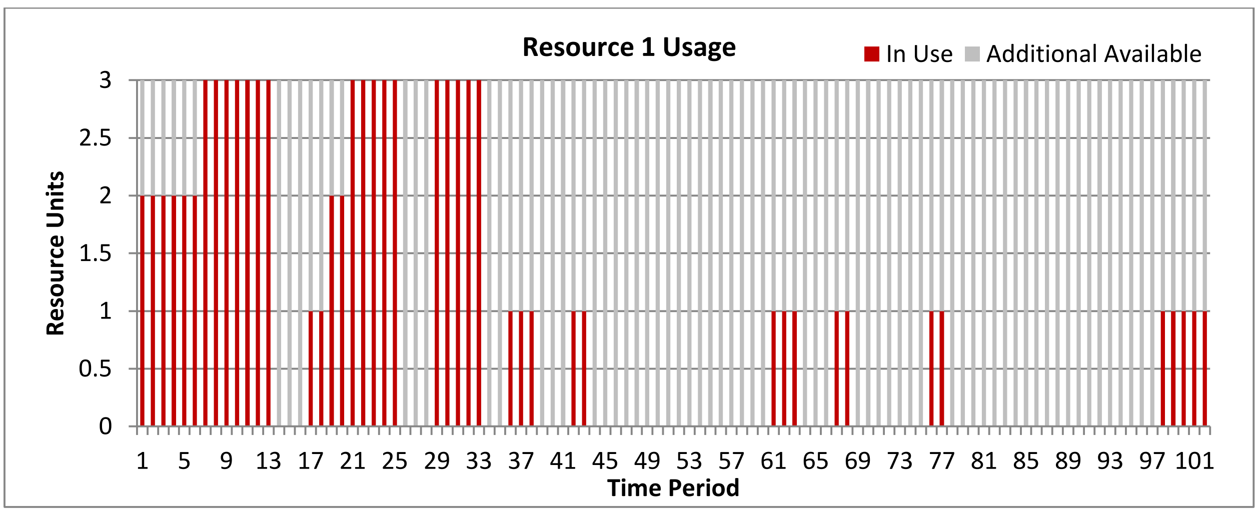

Figure 20 Inspector (Resource 1) Usage

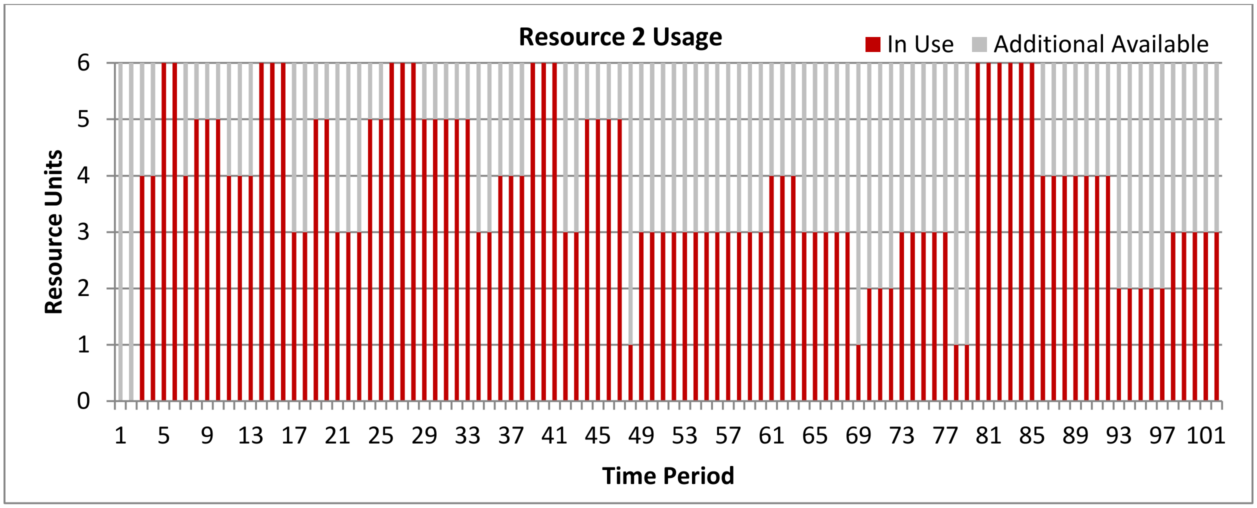

Figure 21 Engineers (Resource 2) Usage

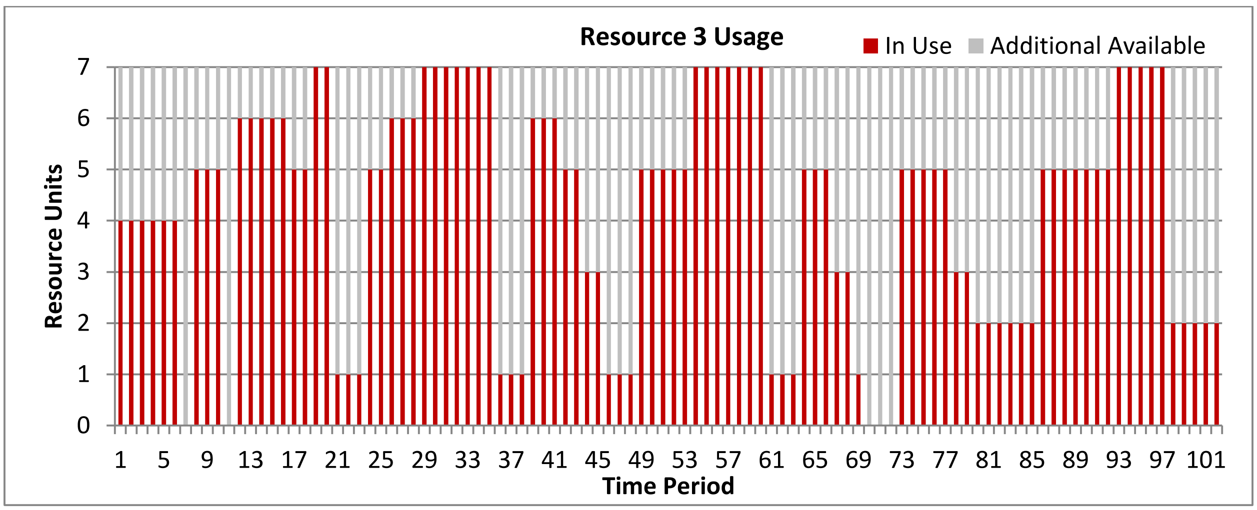

Figure 22 Special Labor (Resource 3) Usage

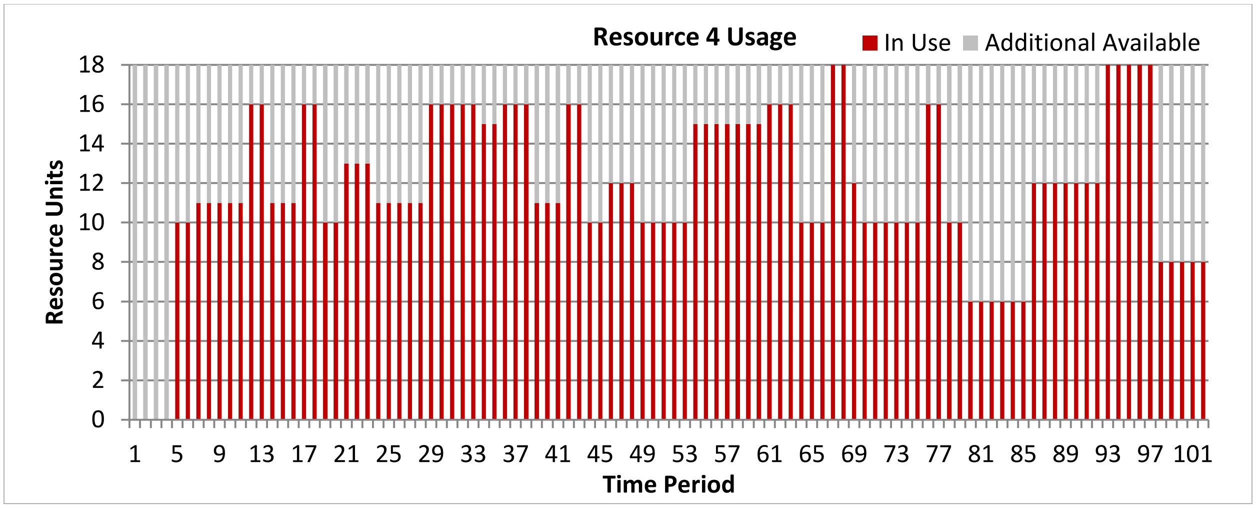

Figure 23 General Labor) (Resource 4 Usage

At the end of time period 13, P6 is recovered and the cost per hour due to unmet demand reduces to \$124000 from \$146000. After that, work on P50 and Pump1 continues in parallel. In time periods 14 to 16, as can be seen in Figure 21, all our engineering units are being employed. Precedence feasible task during these periods are tasks 1, 17, 25 and 39. Tasks, 1, 17 and 25 require engineer units that are not available. Task 39 is prevented from being completed by non-availability of special crew units. In periods, 17 and 18, tasks 12 and 48 of Pump1 and P50 are being completed. Tasks1 and 17 both require 3 special labor units but only two available; task 25 requires 4 engineer units while only 3 are available, and task 39 requires 4 special labor units but available are only 2. At the end of period 18, repair of P50 is completed and the cost per hour due to unmet demand is reduced to \$53000. At the end of period 48, Pump1 gets repaired, and the cost per hour due to unmet demand on the systems is drastically reduced to \$9500 from \$53000. This is because the pump can supply additional water and there is enough connectivity to maintain flow. During periods 39 to 41, tasks 18 and 32 of Pump2 and P4, are continued. These two tasks utilize all engineering units, 6 of 7 available special labor units and 11 of available 18 general labor units. Precedence feasible tasks, task 3, 25 and 40, require more than available general labor or engineering units. Same scenario occurs between time periods 42 and 43 when tasks 19 and 33 of Pump2 and P4 are being completed. At the end of time period 43, P4 is repaired resulting in the cost of unmet demand to only \$26.

At this point, the system is able to meet nearly all demand, but repairs to Pump2, pipes P2 and P48, and the damaged tank remain to be completed. Pump2 is repaired at time 63. At the end of period 68, P48 repair is completed. During time periods 70 to 77, tasks for repair of P2 are completed. Tank repair resumes at time period 76 when required general labor units are available. Tank repair continues until time period 102 when tank repair gets completed and systems become fully functional.

The repair sequence of the damaged elements is provided in Table 10. First row, "Repaired Element", gives the repair sequences. Second row, "Task ID", represents the task required for completing the repair of corresponding element in first row. Third row gives end of time period when the repair of corresponding element is completed. Fourth row represents the reminder cost incurred per hour after the repair of corresponding element. Systemic Impact (SI) of the optimal repair sequence is shown in Figure 24. The SI of the optimal recovery schedule is \$7.26 million. The TRE is \$2.64 million. From Table 10 and Figure 24, it can be clearly observed that as the once one of the pumps is repaired, the impact on the system is drastically reduced. It can also be observed that only few elements, four here, need to be repaired for reducing systemic impact to almost zero. The reason is the high connectivity in the present network.

Table 10 Repair Sequence of damaged elements

Repaired Element	None	P6	P50	PUMP1	P4	PUMP2	P48	P2	P80
Task ID	None	34 to 38	44 to 48	10 to 16	29 to 33	17 to 23	39 to 43	24 to 28	1 to 9
Repair Time Period	None	13	18	38	43	63	68	77	102
H in \$1000	146.682	123.895	53.8419	9.4635	0.02632	0.0183	0.00457	0.000	0

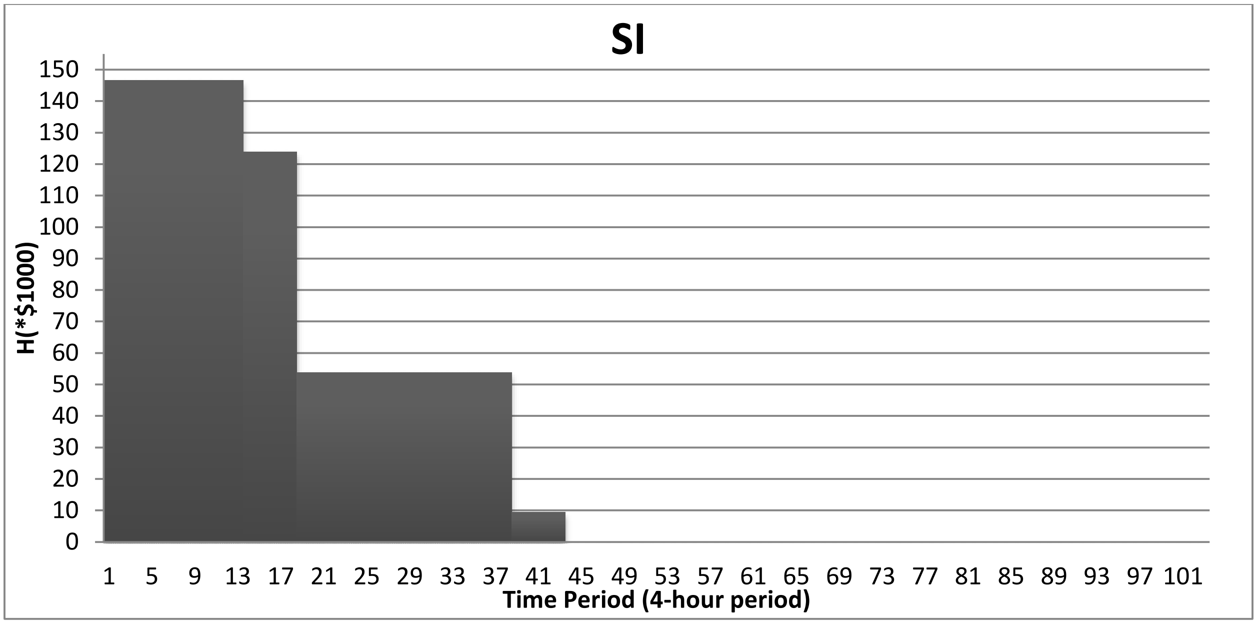

Figure 24 Systemic Impact (SI) for optimal repair sequence

5.3.2 Increased available resources

In this section, we analyze the task scheduling when there are increased resources available. Here we have some different interesting observations as expected. The overall recovery time is reduced to 2/3rd of the time taken in the first experiment. The optimal schedule for this experiment is given in Figure 25. The total time taken for the complete recovery of the system is 69 time periods. The SI, Figure 30, of the optimal recovery schedule is reduced from $7.26 million to $3.5 million. Since, the tasks and their costs remain same as the first experiment, the TRE of this experiment remains $2.64 million. The SI reduction is mainly due to repair of one pump at a much earlier time, facilitated by the increased resource availability. In the first experiment, the repair of first pump was completed at the end of 38th period. In this experiment however, the repair is completed at the end of 30th period, 8 periods earlier.

Figure 25 Optimal Task schedule

Figure 26, Figure 27, Figure 28 and Figure 29 show the usage of Resource 1 (Inspector units), Resource 2 (Engineering units), Resource 3 (Special labor units), and Resource 4 (General labor units), respectively. The optimal schedule starts repair of pipes, P6 and P50, in the first time period. These two pipes are most important for reducing the unmet demand. The increased resource availability is exploited to complete tasks of both pipes in parallel. It would not be possible to start repair of third element, Pump1, at the beginning, because of non-availability of enough inspector units, Figure 26. In time periods 3 and 4, eight out of nine available engineering units are employed for tasks 35 and 45 of P6 and P50; see Figure 27. Hence, at this time period also, work on either of Pumps cannot be started. However, enough resources are available to begin tasks 24 and 29 of P2 and P48. At time period 5, P6 and P50 repair tasks are being carried out, and resources are now available to begin first task, task 10, of Pump1. Task 29 of P4 is also started at the same time, but tasks 25 and 40 of more important pipes, P2 and P48 cannot be carried out because of limited availability of engineering units. Nine engineering units are available; tasks 10, 36 and 46 of Pump1, P6 and P50 utilize 6 of them; working on task 25 or 40 would require 10 engineering units. We cannot perform task 12 because of unavailability of enough general labor units. Because of engineering resource limitations, we can only work on Pump1, P6 and P50. In period 11, task 38 and 48 of P6 and P48 are started. From time period 11, we have some free general labor units available, we start precedence feasible task, task 12 of Pump1 also. At the end of time period 12, we have repaired P6 and P50, and the cost per hour due to unmet demand reduced to \$54000 from \$146000. At time period 13, we continue task 12 of Pump1. Now we have resources for precedence feasible task 13 of Pump1, and task 35 of P2. Our priority is to complete repair of Pump1 and P2 as soon as possible. Therefore, we keep working on these two elements and we have resources for task 40 of P4. We do not have enough general labor units to continue to task 40 of P4, but we do have resources to start work on our next important element, Pump2, so task 17 of Pump2 starts. At the end of period 22, P2 is completely repaired, and the network is recovered to cost per hour due to unmet demand of \$52000. In time periods 23 and 24, we continue work on Pump1 and Pump2; we would also like to work on next task of P4, task 30, but we do not have enough engineering units. Task 41 of P48 could be started at this time period, since it requires much labor for three periods, it will prolong the repair of Pump 2. At the end of time period 24, we have released some engineering units to allow work on P2. In periods 28 to 30, resources are available for tasks, task 16, 19, 41, and 1 of Pump1, Pump2, P48 and tank. Repair of Pump1 is completed at the end of time period 30, and the cost per hour due to unmet demand reduces to $22 from $52000. Virtually the entire demand can be met. The abrupt decrease in unmet demand exemplifies the importance of recovering pumps in water distribution network as soon as possible, provided we have some enough degree of connectivity in the network. The ample connectivity in the system makes it highly reliable; with only a few elements repaired, the system is almost fully recovered.

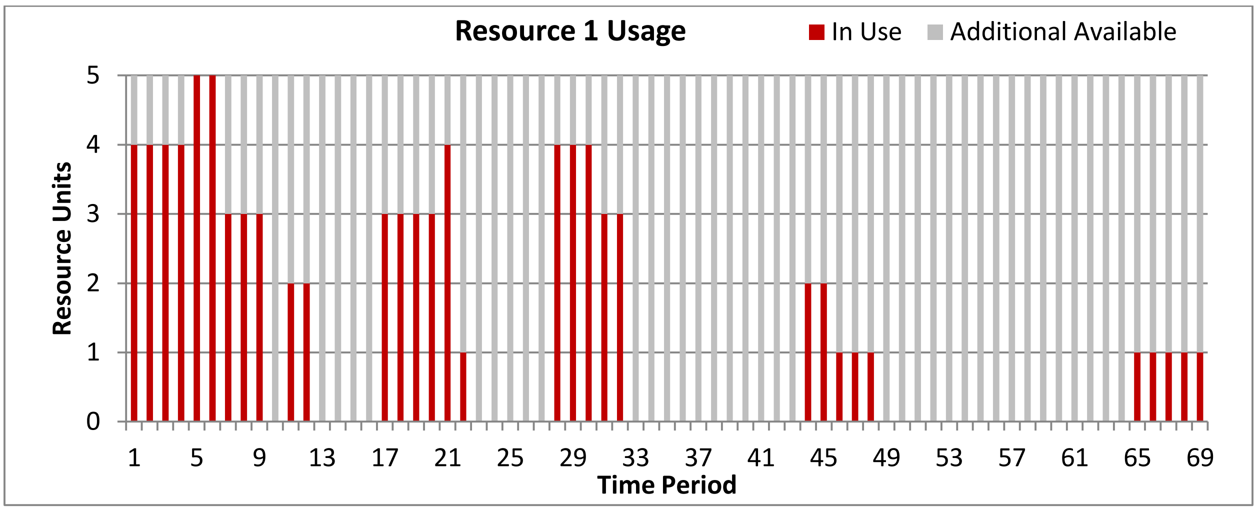

Figure 26 Inspector (Resource 1) Usage

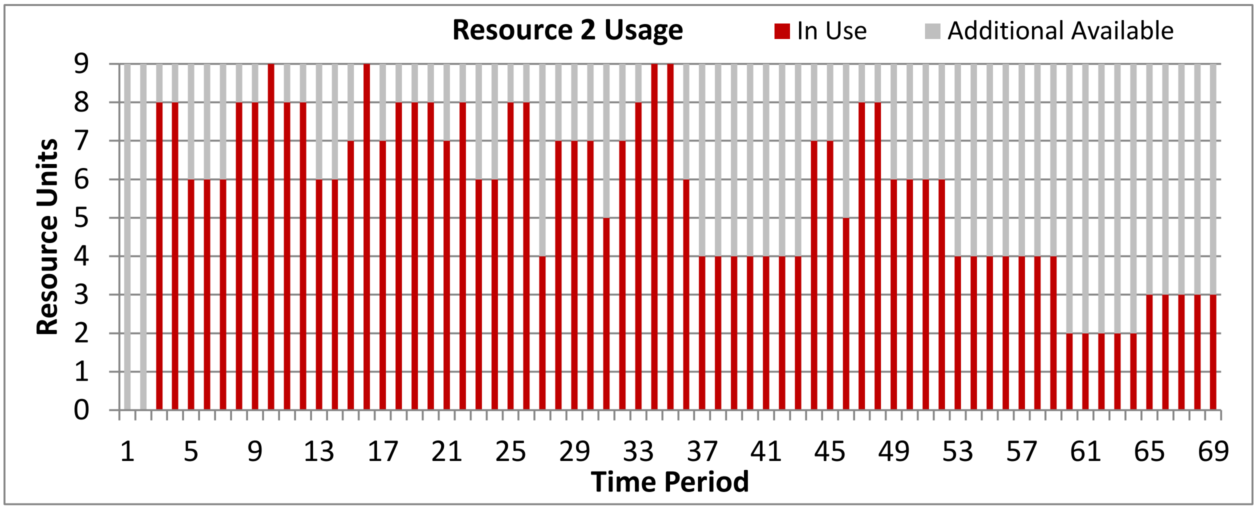

Figure 27 Engineer (Resource 2) Usage

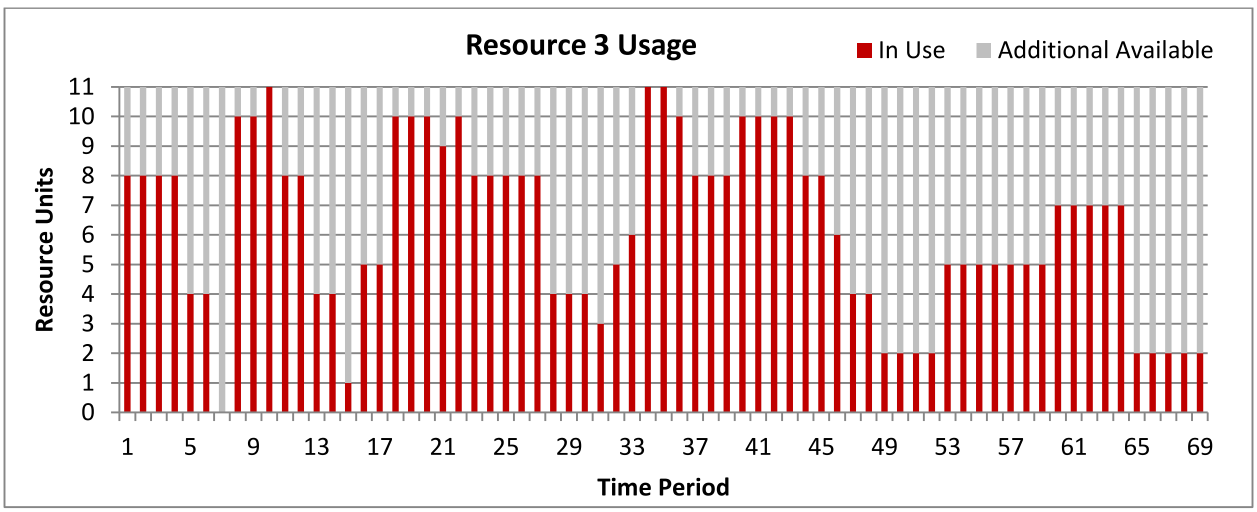

Figure 28 Special Labor (Resource 3) Usage

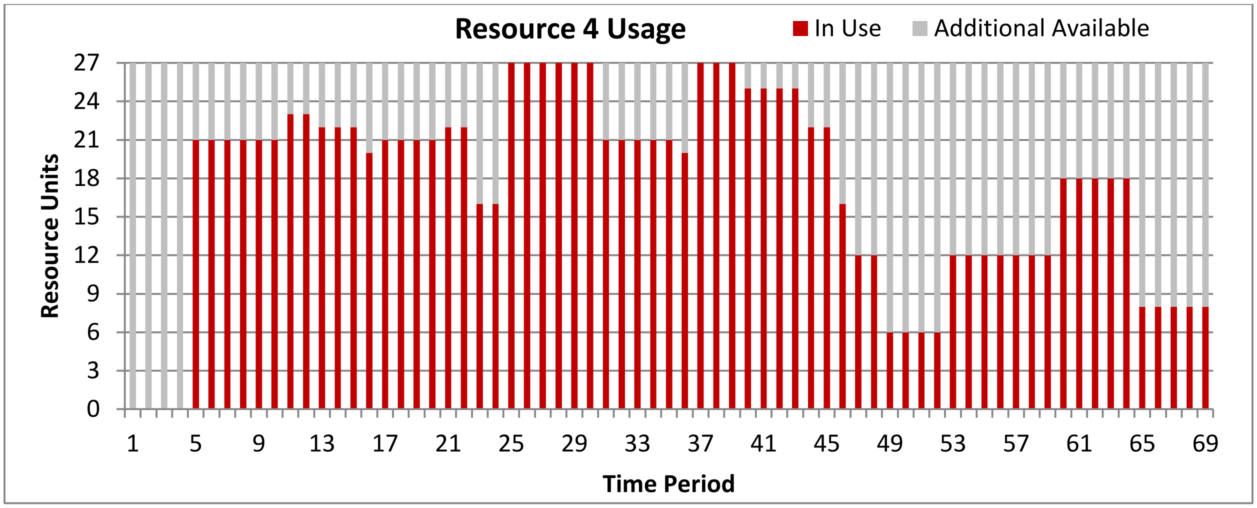

Figure 29 General Labor (Resource 4) Usage

The repair sequence of damaged elements is shown in Table 11.

Table 11 Repair Sequence of damaged elements

Repaired Element	None	P6	P50	P2	PUMP1	P4	PUMP2	P48	P80
Task ID	None	34 to 38	44 to 48	24 to 28	10 to 16	29 to 33	17 to 23	39 to 43	1 to 9
Repair time period	None	12	12	22	30	37	45	47	69
H (*$1000)	146.682	123.89	53.842	52.028	0.0223	0.0222	0.0065	0.000	0

Systemic Impact (SI) of the optimal repair sequence is shown in Figure 30 below.

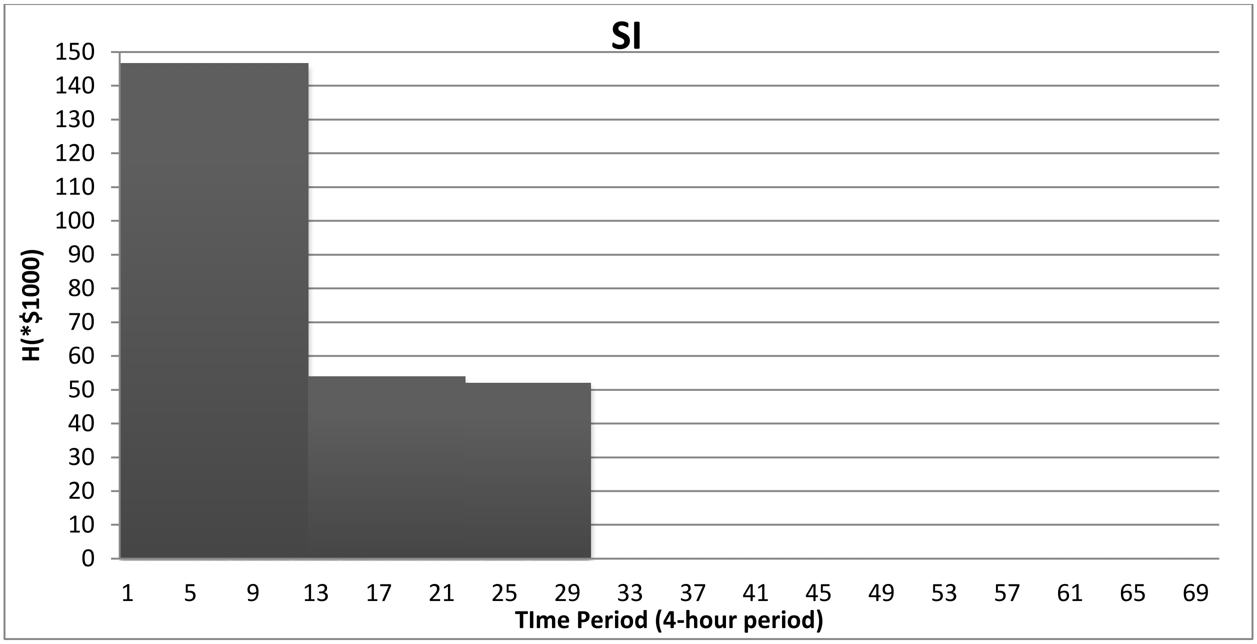

Figure 30 Systemic Impact (SI) for optimal repair sequence

5.3.3 Multi-Mode Tasks

In this section, an additional mode is provided to some tasks. Modes are added to tasks whose duration, resource requirement and cost might provide some flexibility for policy makers to expedite the overall recovery. The modes are added that reduce the duration of tasks. Modes are added to tasks of P2, P6, P50, Pump1 and Pump2 because these elements seem to be important for reducing the SI, see Table 10 and Table 11.

We now compare the optimal schedules of the multi-mode experiment with increased resource experiment. The repair sequence of damaged elements is shown in Table 12. Systemic Impact (SI) of the optimal repair sequence is shown in Figure 31. We observe the overall recovery time is reduced to 68 periods from 69 in the second experiment. We have a slightly different optimal schedule, shown in Figure 32. The SI of the optimal recovery schedule is reduced from $3.5 million to $2.2 million. However, TRE is increased from $2.64 million to $2.91 million. The total cost (SI+TRE) is reduced from $6.16 million to

$5.11 million. We see that providing expedited modes for P6, P50 and Pump1 has reduced the total cost and time of repair significantly.

Table 12 Repair sequence of damaged elements

Repaired Element	None	P6	P50	PUMP1	P2	P4	PUMP2	P48	P80
Repair time period	None	11	11	25	27	35	44	46	68
H(*$1000)	146.682	123.895	53.8417	9.46367	0.0223	0.0222	0.00659	0.000	0

Figure 31 below shows the systemic Impact of the optimal recovery schedule.

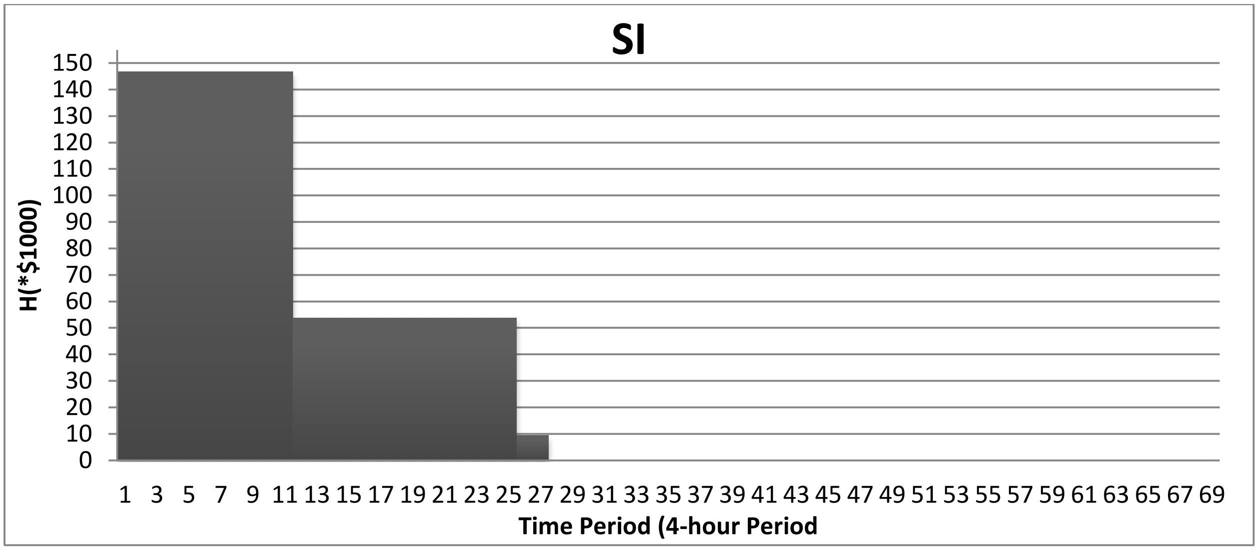

Figure 31 Systemic Impact (SI) for optimal repair sequence

Figure 32 below shows the optimal recover schedule for the multi-mode experiment.

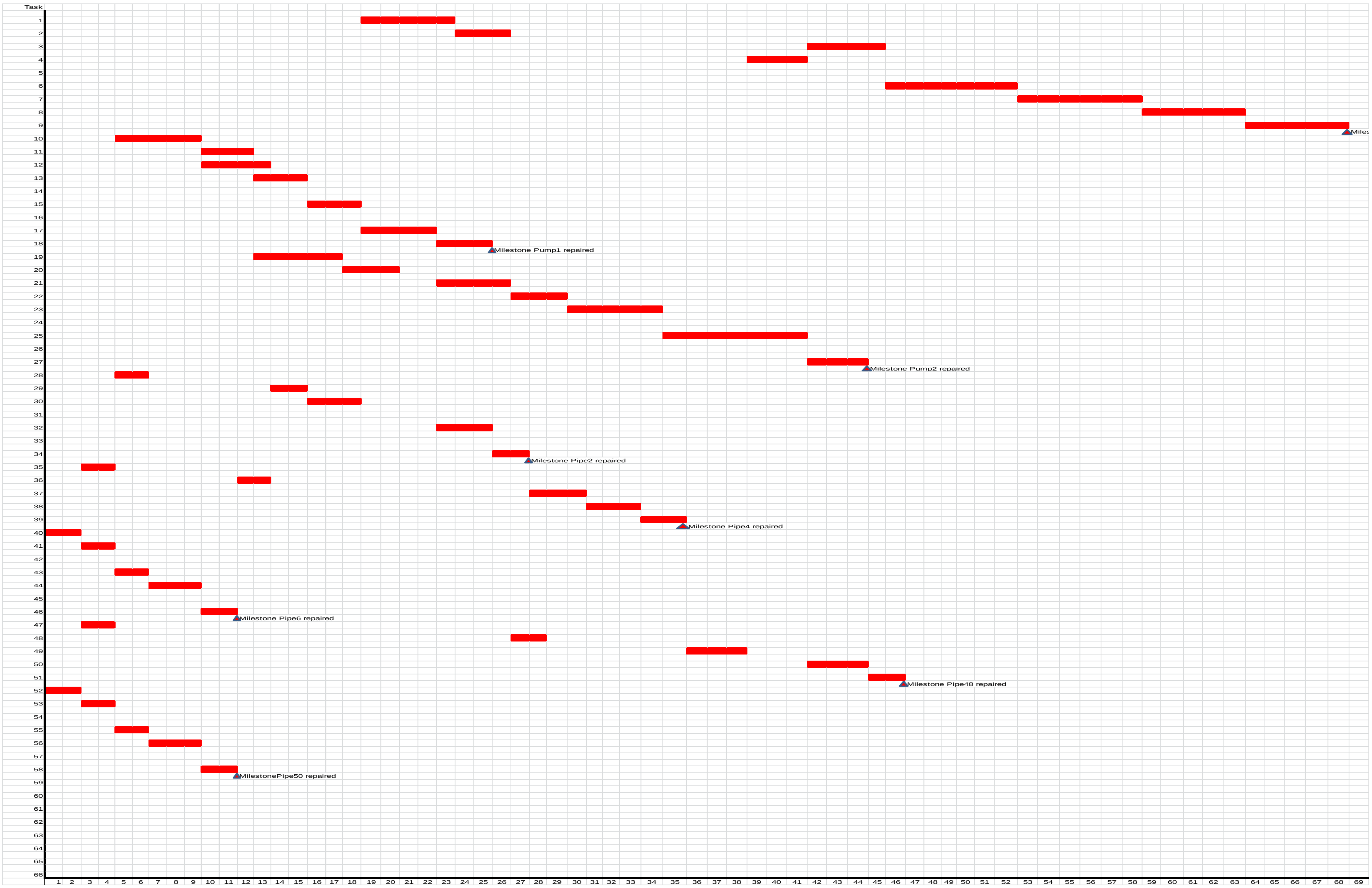

Figure 32 Optimal task schedule

We see in this experiment, the repair of P6 and P50 occur at time period 11, one time period earlier than in 2nd experiment leading to a decrease in SI. It can also be observed that Pump1 repair takes place much earlier. The earlier repair of P6, P50 and Pump1, even though increases TRE, has a significant decrease in the SI, leading to an overall decrease in SI+TRE.

6. Conclusion

In this study a bi-level optimization model is formed for recovery of a disrupted water distribution system. The model schedules the repair tasks in order to minimize the total effect on the system due to disruption. The effect is computed in terms of total repair cost and total systemic impact during the repair process. The model is applied to a small water distribution system with a small damage scenario, and a larger system with a major damage scenario. Validation of the schedule obtained demonstrated that the model scheduled the repair tasks of the small network quite satisfactorily, minimizing the systemic impact as much as one could. We observe that the limited availability of even one resource (engineers in the case of the small system) can have a significant effect on the systemic impact. This limited availability leads to some other resources being unused and overall recovery being delayed.

Application of the model to a larger system helped us to gain more important insights in the recovery of disrupted systems. In the first experiment, having limited resources, the systemic impact of the disruption in the optimal task schedule is quite high, \$7.3 million, and the time taken to recover the system is 102 time periods (about 2.5 weeks). The total cost (SI+TRE) is \$9.9 million. As a consequence of the limited availability of resources, parallel work on different tasks is limited.

With a 50% increase in the available resources in the second experiment, and the same tasks being required for recovery, we observe that systemic impact is reduced by more than 50% and the time to recovery is reduced by about 30%. This exemplifies the importance of making an effort in having sufficient resources available for the recovery of systems. The model allows us to determine when additional resources have diminishing returns. We observe from the resource usage graphs that an increase beyond 50% may not necessarily reduce the cost because the task duration and precedence will become the limiting factors.

In the final experiment, some tasks are provided with one additional mode that might expedite the recovery process of critical elements. We observe that, indeed, providing the additional mode has positive impact on the system recovery. Even though the repair cost incurred increased, the systemic impact and hence, the total recovery cost decreased significantly. The systemic impact in this case reduced to $2.2 million and the total cost went down from $6.16 million to $5.11 million.

Although these experiments have been done on a hypothetical test network, the results have important practical implications. The ability to use the model to create optimal recovery plans under various levels of resource availability is important to public agencies responsible for managing limited resources during recovery after major disruptive events like hurricanes or earthquakes. The recent experience in New York and New Jersey following Hurricane Sandy illustrates how complex the recovery process can be, and how vital it is to be able to assess the effects of varying (and uncertain) level of resource availability for recovery.

The ability to retain flow solutions to different combinations of availability for various network elements also has practical application for re-optimization as knowledge of resource availability changes. Having these flow patterns stored and accessible means that re-optimization with different resource levels can be done faster and more efficiently.

In this study we show that the repair task scheduling of disrupted systems can be done by formulating the optimization task as a bi-level optimization problem. Solution of this optimization problem is an important proof-of-concept demonstration. However, the task scheduling for the larger system tested required considerable computational effort. One of the directions for further research would be to improve the present recovery model by exploring other optimization methods that might reduce the computational requirements.

Another direction for further research is to incorporate the supervisory control and data acquisition (SCADA) system in the network that is subject to damage. SCADA systems provide information about the real time operation of systems and the means of controlling those systems. Damage to the SCADA system itself (whether intentional or the result of a natural hazard) puts the entire system at risk and may render it inoperable. The example analyses in this thesis focus on recovery of the physical network, but tasks and resources required to repair the SCADA system can also be added. Such an extension would change some of the precedence constraints among tasks in the recovery and also modify the milestones at which partial operation of the system becomes feasible.

A third direction for further research is to include decisions on where and in what way to invest in the system to allow it to recover more effectively. Such pre-event investments are vital ways of improving system resilience. However, addressing the investment question is difficult because it is quite uncertain what type of disruptive events may occur in the future and the investments must be evaluated for effectiveness against a wide variety of possible events. Inclusion of pre-event investment also complicates the optimization because it adds a third layer to the bi-level optimization studied here. That is, the recovery actions become dependent on what pre-event investments have been made, and then the system performance is dependent on the recovery actions. Evaluating an investment requires generating a range of possible disruption scenarios, and then optimizing the recovery within each of them, dependent on what investments have been made. This presents an overall optimization of great complexity.

Inclusion of pre-event investments also creates a connection to the general area of asset management and rehabilitation within water distribution systems. The general focus of asset management is to make longer term decisions about investment (where, when and how much) to maintain service within a system that naturally deteriorates over time. Asset management decisions are important in a variety of infrastructure networks, including roads, bridges, gas utilities and communication as well as water networks. Decision support systems for asset management are focused on normal operation of the systems and don’t generally include consideration of resilience against disruption as part of the investment evaluation. However, as concern with resilience becomes a more central part of planning for infrastructure networks, optimization of asset management decisions over time needs to reflect the evaluation of investments designed to improve resilience. This creates another important opportunity for further research.

7. References

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